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An Academic Response to Basel 3.5

References:

[1] Embrechts, P., Puccetti, G., Rüschendorf, L., Wang, R. and A. Beleraj (2014). An Academic Response to Basel 3.5. Risks 2(1), 25-48.

[2] Bignozzi, V., Puccetti, G., and L. Rüschendorf (2014). Reducing model risk via positive and negative dependence assumptions. Insurance Math. Econ., to appear.

[3] Aas, K. and G. Puccetti (2014). Bounds for total economic capital: the DNB case study. Extremes, in press.

talk&papers available on my web: https://sites.google.com/site/giovannipuccetti/

BCBS-Consultative Document, May 2012,
Fundamental Review of the Trading Book (Basel 3.5), p.41 q.8:

8.	What are the likely operational constraints with moving from VaR to ES, including any challenges in delivering robust backtesting, and how might these be best overcome?

An academic response to Basel 3.5

1. Measuring dependence uncertainty: the DNB case
2. Asymptotic equivalence of VaR/ES worst case estimates
3. Adding extra dependence assumptions

DNB risk portfolio used for ICAAP

L_1	L_2	L_3	L_4	L_5	L_6
Credit Risk	Market Risk	Ownership Risk	Operational Risk	Business Risk	Insurance Risk
2.5e06 simulations	2.5e06 simulations	2.5e06 simulations	LogNormal distribution	LogNormal distribution	LogNormal distribution

$L_d^+ = L_1 + \cdots + L_d$ total loss exposure (for DNB: d=6)

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Basel II(I) requirement: compute and reserve based on

$\mathrm{VaR}_\alpha(L_d^+)$ or $\mathrm{ES}_\alpha(L_d^+)$

Value-at-Risk (VaR)

$$\mathrm{VaR}_\alpha(L_d^+) = \inf\{x \in \mathbb{R} : F_{L_d^+}(x) > \alpha\},\ \alpha \in (0,1).$$

$$P(L_d^+ > \mathrm{VaR}_\alpha(L_d^+)) \leq 1 - \alpha.$$

Expected Shortfall (ES)

$$\mathrm{ES}_\alpha(L_d^+) = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_q(L_d^+)\,dq,\ \ \alpha \in (0,1).$$

$$\mathrm{ES}_\alpha(L_d^+) = E[L_d^+ | L_d^+ > \mathrm{VaR}_\alpha(L^+)],\ \ \text{if } L_d^+ \text{ is continuous.}$$

ES is a coherent risk measure

$$\mathrm{ES}_\alpha(L_d^+) \leq \mathrm{ES}_\alpha^+(L_d^+) := \sum_{i=1}^{d} \mathrm{ES}_\alpha(L_i)$$

VaR fails to be subadditive

$$\mathrm{VaR}_\alpha(L_d^+) > \mathrm{VaR}_\alpha^+(L_d^+) := \sum_{i=1}^{d} \mathrm{VaR}_\alpha(L_i).$$

comonotonic dependence
(maximal correlation)

General problem

L_1	L_2	L_3	L_4	L_5	L_6
Credit Risk	Market Risk	Ownership Risk	Operational Risk	Business Risk	Insurance Risk
2.5e06 simulations	2.5e06 simulations	2.5e06 simulations	LogNormal distribution	LogNormal distribution	LogNormal distribution

one period risks with statistically estimated marginals

DU-spread for VaR

superadditive models

$\underline{\mathrm{VaR}}_\alpha(L_d^+)$ $\sum_{i=1}^d \mathrm{VaR}_\alpha(L_i)$ $\overline{\mathrm{VaR}}_\alpha(L_d^+)$

DU-spread for ES

$\underline{\mathrm{ES}}_\alpha(L_d^+)$ $\overline{\mathrm{ES}}_\alpha(L_d^+)$

General problem

L_1	L_2	L_3	L_4	L_5	L_6
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one period risks with statistically estimated marginals

and unknown dependence structure

DU-spread for VaR

superadditive models

$\underline{\mathrm{VaR}}_\alpha(L_d^+)$ $\quad \sum_{i=1}^{d} \mathrm{VaR}_\alpha(L_i)$ $\quad \overline{\mathrm{VaR}}_\alpha(L_d^+)$

$$\overline{\mathrm{VaR}}_\alpha(L_d^+) := \sup\{\mathrm{VaR}_\alpha(L_1 + \cdots + L_d) : L_i \sim F_i, 1 \le i \le d\},$$

$$\underline{\mathrm{VaR}}_\alpha(L_d^+) := \inf\{\mathrm{VaR}_\alpha(L_1 + \cdots + L_d) : L_i \sim F_i, 1 \le i \le d\}.$$

DU-spread for ES

$\underline{\mathrm{ES}}_\alpha(L_d^+)$ $\quad \overline{\mathrm{ES}}_\alpha(L_d^+)$

L_1	L_2	L_3	L_4	L_5	L_6
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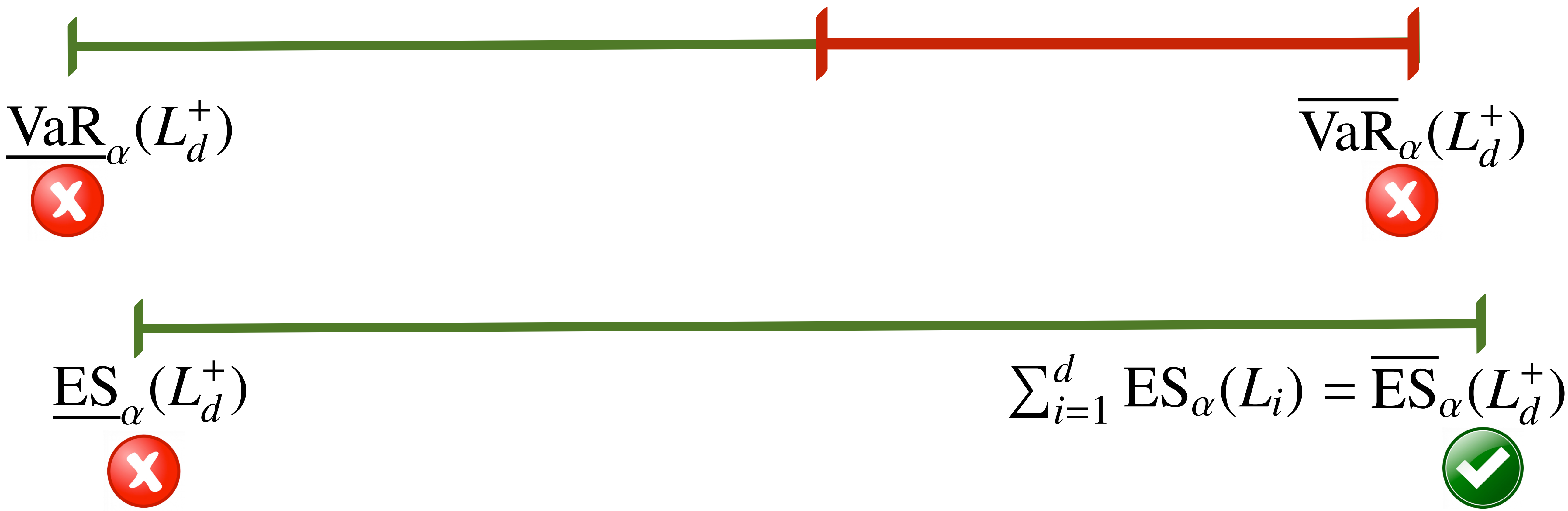
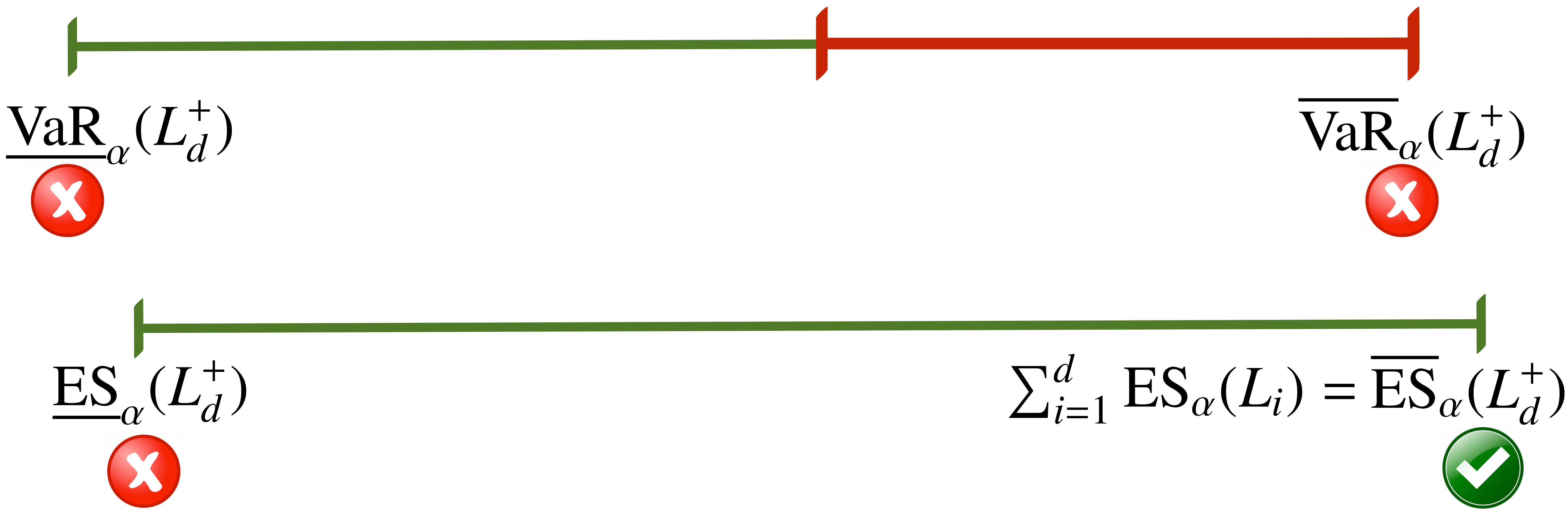
DU-spread for ES

$\underline{\mathrm{ES}}_\alpha(L_d^+)$ $\sum_{i=1}^d \mathrm{ES}_\alpha(L_i) = \overline{\mathrm{ES}}_\alpha(L_d^+)$



For general inhomogenous marginals, there does not exist an analytical tool to compute .

How can we compute the bounds?



For general inhomogenous marginals, there does not exist an analytical tool to compute .

Then use the Rearrangement Algorithm; see [3] for a step-by-step implementation.

PARETO(2) MARGINALS AND α=0.99

	1	2	3	Σ
1	9.00000	9.00000	9.00000	27.0000
2	9.17095	9.17095	9.17095	27.5129
3	9.35098	9.35098	9.35098	28.0530
4	9.54093	9.54093	9.54093	28.6228
5	9.74172	9.74172	9.74172	29.2252
6	9.95445	9.95445	9.95445	29.8634
7	10.18034	10.18034	10.18034	30.5410
8	10.42080	10.42080	10.42080	31.2624
9	10.67748	10.67748	10.67748	32.0325
10	10.95229	10.95229	10.95229	32.8569
11	11.24745	11.24745	11.24745	33.7423
12	11.56562	11.56562	11.56562	34.6969
13	11.90994	11.90994	11.90994	35.7298
14	12.28422	12.28422	12.28422	36.8527
15	12.69306	12.69306	12.69306	38.0792
16	13.14214	13.14214	13.14214	39.4264
17	13.63850	13.63850	13.63850	40.9155
18	14.19109	14.19109	14.19109	42.5733
19	14.81139	14.81139	14.81139	44.4342
20	15.51446	15.51446	15.51446	46.5434
21	16.32051	16.32051	16.32051	48.9615
22	17.25742	17.25742	17.25742	51.7723
23	18.36492	18.36492	18.36492	55.0948
24	19.70197	19.70197	19.70197	59.1059
25	21.36068	21.36068	21.36068	64.0820
26	23.49490	23.49490	23.49490	70.4847
27	26.38613	26.38613	26.38613	79.1584
28	30.62278	30.62278	30.62278	91.8683
29	37.72983	37.72983	37.72983	113.1895
30	53.77226	53.77226	53.77226	161.3168
Σ	494.99920	494.99920	494.99920	NA

ORDERED MATRIX

	1	2	3	
[1,]	16.32051	14.19109	13.63850	44.1501
[2,]	13.14214	17.25742	14.19109	44.5906
[3,]	12.28422	12.69306	19.70197	44.6793
[4,]	18.36492	13.63850	12.69306	44.6965
[5,]	11.56562	18.36492	14.81139	44.7419
[6,]	19.70197	13.14214	11.90994	44.7540
[7,]	12.69306	14.81139	17.25742	44.7619
[8,]	17.25742	11.24745	16.32051	44.8254
[9,]	11.90994	11.56562	21.36068	44.8362
[10,]	11.24745	21.36068	12.28422	44.8924
[11,]	21.36068	12.28422	11.24745	44.8924
[12,]	13.63850	19.70197	11.56562	44.9061
[13,]	15.51446	16.32051	13.14214	44.9771
[14,]	14.81139	11.90994	18.36492	45.0862
[15,]	10.95229	10.67748	23.49490	45.1247
[16,]	10.67748	23.49490	10.95229	45.1247
[17,]	23.49490	10.95229	10.67748	45.1247
[18,]	14.19109	15.51446	15.51446	45.2200
[19,]	26.38613	10.42080	10.18034	46.9873
[20,]	10.42080	10.18034	26.38613	46.9873
[21,]	10.18034	26.38613	10.42080	46.9873
[22,]	30.62278	9.74172	9.95445	50.3190
[23,]	9.95445	30.62278	9.74172	50.3190
[24,]	9.74172	9.95445	30.62278	50.3190
[25,]	9.54093	37.72983	9.35098	56.6217
[26,]	37.72983	9.35098	9.54093	56.6217
[27,]	9.35098	9.54093	37.72983	56.6217
[28,]	9.00000	9.17095	53.77226	71.9432
[29,]	9.17095	53.77226	9.00000	71.9432
[30,]	53.77226	9.00000	9.17095	71.9432
[31,]	494.99920	494.99920	494.99920	NA

With N=10^5, we obtain the first three decimals of $\overline{\mathrm{VaR}}_\alpha(L_3^+) = 45.9898$ in 0.2 sec.

DNB risk portfolio (figures in million NOK)

L_1	L_2	L_3	L_4	L_5	L_6
Credit Risk	Market Risk	Ownership Risk	Operational Risk	Business Risk	Insurance Risk
2.5e06 simulations	2.5e06 simulations	2.5e06 simulations	LogNormal distribution	LogNormal distribution	LogNormal distribution

quantile level used: $\alpha = 99.97\%$

62,156.4 — $\underline{\mathrm{VaR}}_\alpha(L_d^+)$

93,152.7 — $\sum_{i=1}^d \mathrm{VaR}_\alpha(L_i)$

105,878.2 — $\overline{\mathrm{VaR}}_\alpha(L_d^+)$

$$\frac{\overline{\mathrm{VaR}}_\alpha(L_d^+)}{\sum_{i=1}^d \mathrm{VaR}_\alpha(L_i)} = 1.136$$

The worst diversification ratio: definition; see[3].

The worst superadditivity (or diversification) ratio for L_d^+ is defined as

$$\overline{\Delta}_\alpha(L_d^+) := \frac{\overline{\mathrm{VaR}}_\alpha(L_d^+)}{\mathrm{VaR}_\alpha^+(L_d^+)} = \frac{\overline{\mathrm{VaR}}_\alpha(L_d^+)}{\sum_{i=1}^d \mathrm{VaR}_\alpha(L_i)}$$

← worst-possible dependence (numerator)

← comotonic dependence (denominator)

The worst superadditivity (or diversification) ratio for L_d^+ is defined as

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← worst-possible dependence (numerator)

← comotonic dependence (denominator)

Examples:

- $\overline{\Delta}_\alpha(L_d^+) = 1$: the aggregate position is always less risky than the sum of the marginal exposures. Examples: $(L_1, \ldots, L_d)$ has a <u>multivariate</u> Gaussian or <u>multivariate</u> Student's t (in general elliptical) distribution.

- $\overline{\Delta}_\alpha(L_d^+) > 1$: superadditivity of VaR. It typically occurs with heavy-tailed and/or skew marginals and/or non-elliptical portfolios.

Explicit upper bound in the homogeneous case (all risks have df F) :

$$\overline{\Delta}_\alpha(L_d^+) := \frac{\overline{\mathrm{VaR}}_\alpha(L_d^+)}{\mathrm{VaR}_\alpha^+(L_d^+)} \leq \frac{\overline{\mathrm{ES}}_\alpha(L_d^+)}{\mathrm{VaR}_\alpha^+(L_d^+)} = \frac{d\mathrm{ES}_\alpha(L_1)}{d\mathrm{VaR}_\alpha(L_1)} = \frac{\mathrm{ES}_\alpha(L_1)}{\mathrm{VaR}_\alpha(L_1)}.$$

Explicit upper bound in the homogeneous case (all risks have df F) :

$$\overline{\Delta}_\alpha(L_d^+) := \frac{\overline{\mathrm{VaR}}_\alpha(L_d^+)}{\mathrm{VaR}_\alpha^+(L_d^+)} \leq \frac{\overline{\mathrm{ES}}_\alpha(L_d^+)}{\mathrm{VaR}_\alpha^+(L_d^+)} = \frac{d\mathrm{ES}_\alpha(L_1)}{d\mathrm{VaR}_\alpha(L_1)} = \frac{\mathrm{ES}_\alpha(L_1)}{\mathrm{VaR}_\alpha(L_1)}.$$

Theorem: Under some general marginal conditions (including all the continuous distributional models used in QRM) + LOSSES WITH FINITE MEAN, we have

$$\lim_{d\to\infty} \overline{\Delta}_\alpha(L_d^+) = \frac{\mathrm{ES}_\alpha(L_1)}{\mathrm{VaR}_\alpha(L_1)} \qquad \text{(homogeneous case)}$$

α	$\theta = 1.1$	$\theta = 1.5$	$\theta = 2$	$\theta = 3$	$\theta = 4$
0.99	11.154337	3.097350	2.111111	1.637303	1.487492
0.995	11.081599	3.060242	2.076091	1.603135	1.454080
0.999	11.018773	3.020202	2.032655	1.555556	1.405266

Values for the limit for Pareto(θ) distributions

α	$\theta = 0.5$	$\theta = 1$	$\theta = 1.5$	$\theta = 2$	$\theta = 2.5$
0.99	1.200364	1.487037	1.920334	2.621718	3.858599
0.995	1.184949	1.443519	1.823195	2.415980	3.415242
0.999	1.158988	1.372433	1.670393	2.107238	2.787941

Values for the limit for LogNormal(0,θ) distributions

α	$\theta = 0.5$	$\theta = 1$	$\theta = 1.5$	$\theta = 2$	$\theta = 2.5$
0.99	1.217147	1.217147	1.217147	1.217147	1.217147
0.995	1.188739	1.188739	1.188739	1.188739	1.188739
0.999	1.144765	1.144765	1.144765	1.144765	1.144765

Values for the limit for Exponential(θ) distributions

Model uncertainty: general risk portfolio with infinite mean

What if the losses have infinite mean?

Under some general marginal conditions, we have that

$$\lim_{d \to \infty} \overline{\Delta}_\alpha(L_d^+) = \infty.$$

This means that the VaR for a sum can be arbitrarily large with respect to the corresponding VaR estimate for comonotonic risks.

2. Asymptotic equivalence of VaR/ES worst case estimates

DNB risk portfolio

L_1	L_2	L_3	L_4	L_5	L_6
Credit Risk	Market Risk	Ownership Risk	Operational Risk	Business Risk	Insurance Risk
2.5e06 simulations	2.5e06 simulations	2.5e06 simulations	LogNormal distribution	LogNormal distribution	LogNormal distribution

quantile level used: $\alpha = 99.97\%$

62,156.4 — $\underline{\mathrm{VaR}}_\alpha(L_d^+)$

93,152.7 — $\sum_{i=1}^{d} \mathrm{VaR}_\alpha(L_i)$

105,878.2 — $\overline{\mathrm{VaR}}_\alpha(L_d^+)$

74,354.7 — $\underline{\mathrm{ES}}_\alpha(L_d^+)$

110,588.8 — $\overline{\mathrm{ES}}_\alpha(L_d^+)$

In general, we have

$$\frac{\overline{\mathrm{VaR}}_\alpha(L_d^+)}{\overline{\mathrm{ES}}_\alpha(L_d^+)} \leq 1.$$

Equivalence of worst VaR and ES estimates

In general, we have

$$\frac{\overline{\mathrm{VaR}}_\alpha(L_d^+)}{\overline{\mathrm{ES}}_\alpha(L_d^+)} \leq 1.$$

Theorem: Under some general marginal conditions (including all the continuous inhomogeneous models used in QRM) + LOSSES WITH FINITE MEAN we have

$$\lim_{d\to\infty} \frac{\overline{\mathrm{VaR}}_\alpha(L_d^+)}{\overline{\mathrm{ES}}_\alpha(L_d^+)} = 1.$$

Equivalence of worst VaR and ES estimates

d	$\overline{\mathrm{VaR}}_{0.999}\left(\sum_{i=1}^{d} L_i\right)$	$\overline{\mathrm{ES}}_{0.999}\left(\sum_{i=1}^{d} L_i\right)$	ratio
3	640.0679	668.7629	1.0448
10	2225.8490	2229.2100	1.0015
50	11146.0300	11146.0500	1.0000
100	22292.1000	22292.1000	1.0000

Sum of d LogNormal(2,1) marginals.

d	$\overline{\mathrm{VaR}}_{0.999}\left(\sum_{i=1}^{d} L_i\right)$	$\overline{\mathrm{ES}}_{0.999}\left(\sum_{i=1}^{d} L_i\right)$	ratio
3	186.49	237.33	1.2726
9	687.09	711.98	1.0362
30	2370.39	2373.26	1.0012
99	7831.72	7831.75	1.0000

Sum of d different Pareto, LogNormal, Exponential marginals.

- The limit is evident also for relatively small dimensions;

- Important consequences wrt the forthcoming Basel 3+ accords.

3. Adding extra dependence assumptions

WORST VAR SCENARIO

	1	2	3	
[1,]	16.32051	14.19109	13.63850	44.1501
[2,]	13.14214	17.25742	14.19109	44.5906
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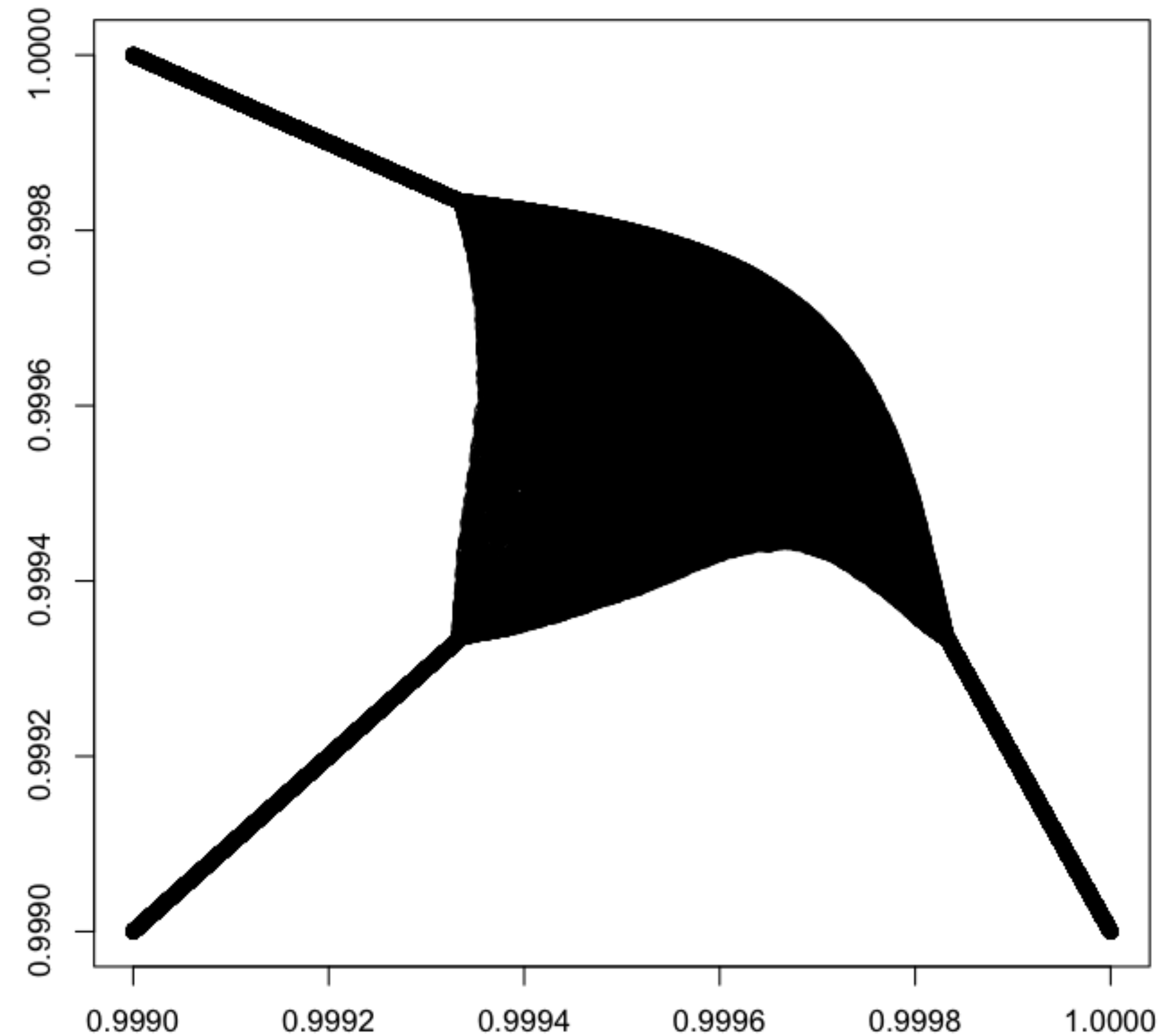
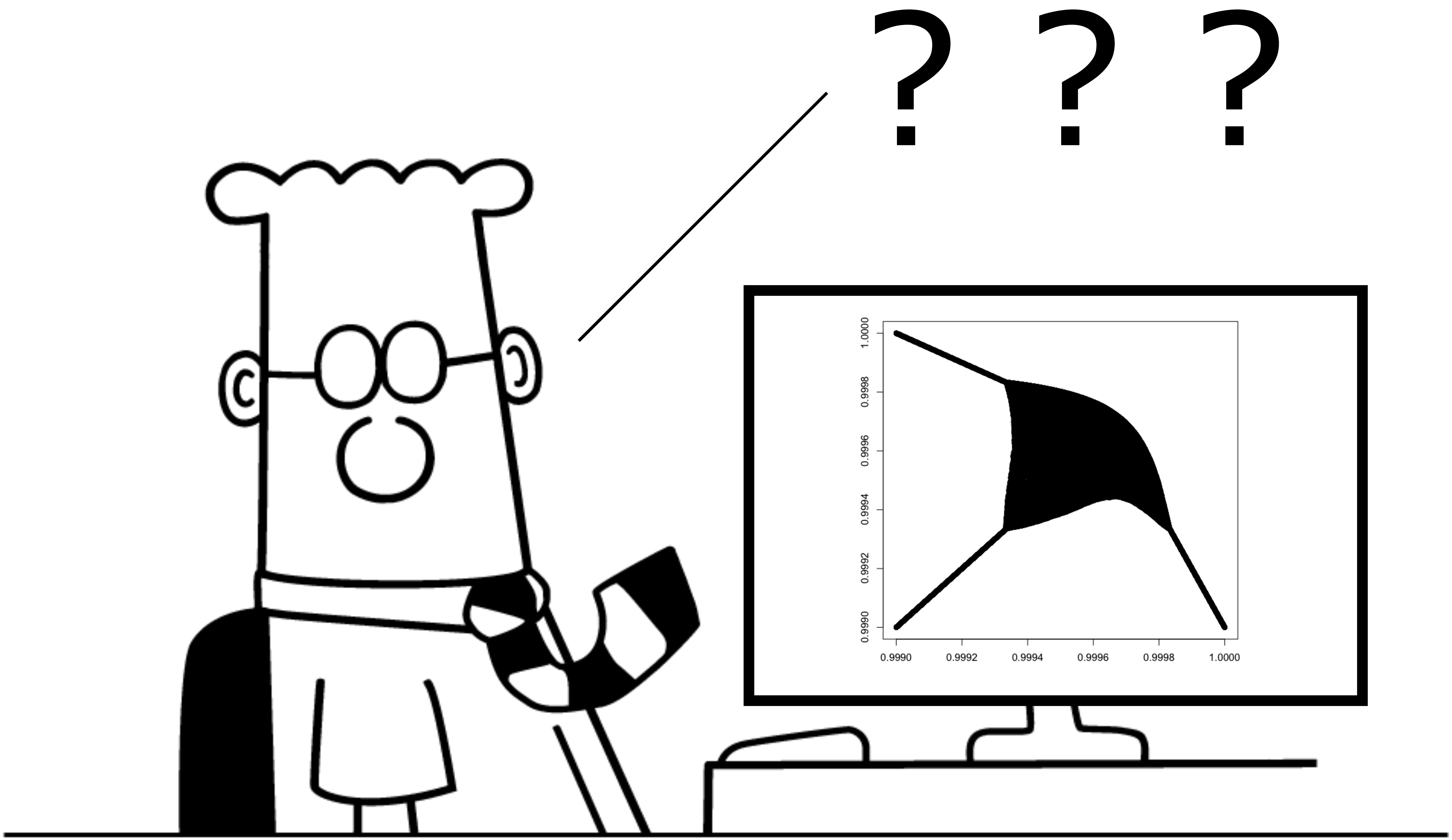


2d projections to [0.999,1]^2 of the support of the 3d-copula merging the upper 99.9%-tails of the three Pareto(2) distributed random variables maximising the 99.9%-VaR of their sum.
The black area represents a completely mixable part, see [1].


? ? ?
1.0000
0.9998
0.9996
0.9994
0.9992
0.9990
0.9990
0.9992
0.9994
0.9996
0.9998
1.0000

In quantitative risk management, the components of a risk portfolio often have some positive dependence structure.

For $\boldsymbol{X}$ and $\boldsymbol{Y}$ in $\mathbb{R}^d$, we define the *concordance order* $\boldsymbol{Y} \leq_{\text{co}} \boldsymbol{X}$, if both

$$\overline{F}_{\boldsymbol{Y}}(\boldsymbol{x}) \leq \overline{F}_{\boldsymbol{X}}(\boldsymbol{x}) \quad \text{and} \quad F_{\boldsymbol{Y}}(\boldsymbol{x}) \leq F_{\boldsymbol{X}}(\boldsymbol{x})$$

hold for all $\boldsymbol{x} \in \mathbb{R}^d$.

The concordance order $\boldsymbol{Y} \leq_{\text{co}} \boldsymbol{X}$ implies

$$\text{Cov}(Y_i, Y_j) \leq \text{Cov}(X_i, X_j); \quad \rho_S(Y_i, Y_j) \leq \rho_S(X_i, X_j); \quad \tau(Y_i, Y_j) \leq \tau(X_i, X_j);$$

where ρ_S is Spearman's and τ is Kendall's rank correlation coefficient.

<u>Typical assumption: POD risks, i.e.</u> $\boldsymbol{L}^{\perp\!\!\!\perp} \leq_{\text{co}} \boldsymbol{L}$

Adding positive dependence I; see [2]

Our assumptions: $\alpha = 99.9\%$

comonotonicity within

$$L_1, L_2, L_3, L_4 \perp\!\!\!\perp L_5, L_6, L_7, L_8$$

comonotonicity within

Pareto(2) marginals

independence between

Exp(1) marginals

Our assumptions: $\alpha = 99.9\%$

comonotonicity within

comonotonicity within

$$L_1, L_2, L_3, L_4 \perp\!\!\!\perp L_5, L_6, L_7, L_8 \qquad \leq_{\text{co}} (L_1, \ldots, L_8)$$

Pareto(2) marginals

Exp(1) marginals

independence between

Adding positive dependence I; see [2]

Our assumptions: $\alpha = 99.9\%$

comonotonicity within

comonotonicity within

$$L_1, L_2, L_3, L_4 \perp\!\!\!\perp L_5, L_6, L_7, L_8 \qquad \leq_{co} (L_1, \ldots, L_8)$$

Pareto(2) marginals

independence between

Exp(1) marginals

DU-S with marginals info only

30.62 — $\underline{\mathrm{VaR}}_\alpha(L_d^+)$

150.12 — $\sum_{i=1}^d \mathrm{VaR}_\alpha(L_i)$

248.24 — $\overline{\mathrm{VaR}}_\alpha(L_d^+)$

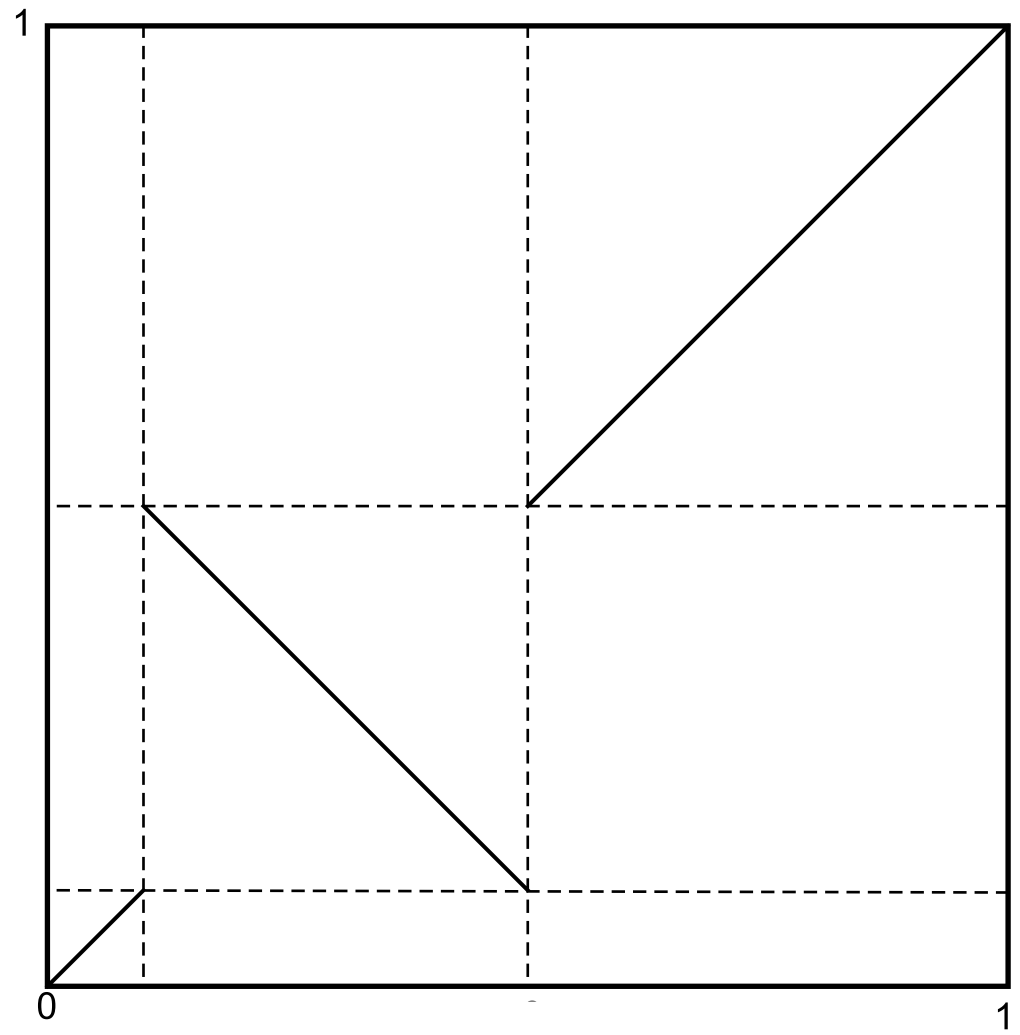
DU-S with marginals info
AND $\leq_{co}$ assumption

122.49 — $\underline{\mathrm{VaR}}'_\alpha$

150.12 — $\sum_{i=1}^d \mathrm{VaR}_\alpha(L_i)$

205.27 — $\overline{\mathrm{VaR}}'_\alpha$

Why positive dependence does not help



This copula is POD!

Positive dependence assumption:

$$\text{If } (L_1^{\perp\!\!\!\perp}, L_2^{\perp\!\!\!\perp}) \leq_{\text{co}} (L_1, L_2) \text{ then } \mathrm{ES}_\alpha(L_1^{\perp\!\!\!\perp} + L_2^{\perp\!\!\!\perp}) \leq \mathrm{ES}_\alpha(L_1 + L_2)$$

Positive dependence assumption:

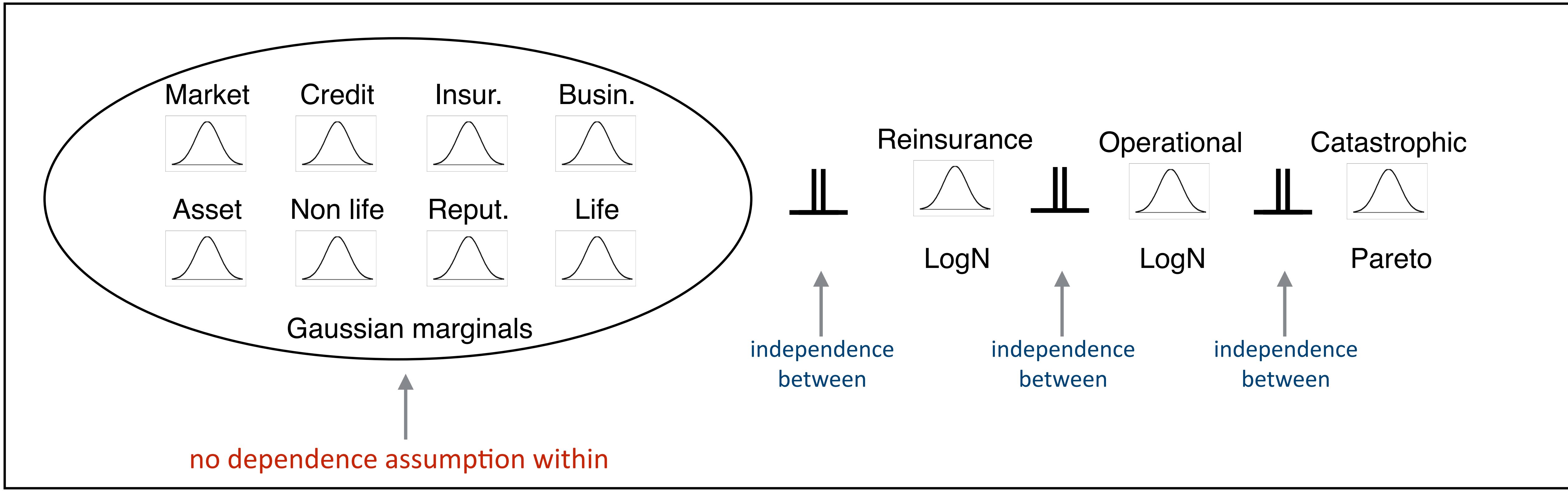
$$\text{If } (L_1^{\perp\!\!\!\perp}, L_2^{\perp\!\!\!\perp}) \leq_{\text{co}} (L_1, L_2) \text{ then } \text{ES}_\alpha(L_1^{\perp\!\!\!\perp} + L_2^{\perp\!\!\!\perp}) \leq \text{ES}_\alpha(L_1 + L_2)$$

Negative dependence assumption:

$$\text{If } (L_1, L_2) \leq_{\text{co}} (L_1^{\perp\!\!\!\perp}, L_2^{\perp\!\!\!\perp}) \text{ then } \text{ES}_\alpha(L_1 + L_2) \leq \text{ES}_\alpha(L_1^{\perp\!\!\!\perp} + L_2^{\perp\!\!\!\perp})$$

These ordering results can be generalized to arbitrary dimensions and law invariant, convex risk measure using the weakly conditional increasing in sequence order or the supermodular order between vectors; see a variety of examples in [2].

Our assumptions: $\alpha = 99.9\%$



121.5 — DU-S with marginals info only — 304.63 — 367.70

$\underline{\mathrm{VaR}}_\alpha(L_d^+)$ $\quad \sum_{i=1}^d \mathrm{VaR}_\alpha(L_i)$ $\quad \overline{\mathrm{VaR}}_\alpha(L_d^+)$

VaR upper bound with marginal info AND independent subgroups $\quad \overline{\mathrm{VaR}}'_\alpha = 256.04$

Final remarks and warnings; see [3]

- Adding positive dependence info is not useful to reduce worst bounds: One should instead assume some independence/negative dependence structure in order to reduce the upper bound on a risk.

- VaR vs ES: If you take a worst-case perspective, they are asymptotically equivalent.

- Superadditivity of VaR: We have analytical and numerical techniques available for the computation of VaR/ES uncertainty range.

- There's more under the top of the iceberg: The risk assessment of a multivariate bank portfolio cannot be reduced to a single VaR number. The superadditivity ratio and the VaR/ES uncertainty range might help to assess the implied model risk.

Forthcoming event and some initiatives on dependence

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