Traffic Dynamics at Intersections Subject to Random Misperception

Volker Berkhahn, Marcel Kleiber, Johannes Langner, Chris Timmermann, Stefan Weber

***Abstract*—Traffic accidents cause harm to the society. Future technology in autonomous vehicles is expected to eliminate the human factor as one important cause of failure. However, in the near future, autonomous vehicles and human drivers will coexist and downside risk still needs to be tolerated in exchange for mobility. Unsignalized intersections are particularly prone to accidents, as lots of potential conflicts between traffic participants occur. Motorists need to anticipate these on the basis of their perception of the environment and react accordingly. Yet, perceptional errors affect human drivers, and it is important to understand their impact on traffic safety and traffic efficiency. We develop a microscopic model of traffic dynamics at single-lane unsignalized intersections subject to random misperception that may cause accidents. Perceptional errors can be modeled by stochastic processes, e.g., Ornstein-Uhlenbeck processes. We present suitable simulation techniques and characterize the behavior of the traffic system in various case studies. We discuss the impact of errors and safety margins on traffic flow, the number of accidents, and the number of collided vehicles. In terms of perception errors, we consider both homogeneous and heterogeneous traffic participants, reflecting the coexistence of human drivers and autonomous vehicles. The model captures the real-world tradeoff between safety and efficiency for potential future traffic systems.**

***Index Terms*—Autonomous vehicles, perception errors, microscopic traffic models, random ordinary differential equations, accidents, traffic flow.**

I. Introduction

THE self-organization of traffic is a highly complex phenomenon. Traffic flow is distorted by accidents which are often triggered by errors in perception or judgement of traffic participants. It seems plausible that in a future world of autonomous vehicles improved technology will substantially reduce, but not completely eliminate traffic accidents (cf., e.g., [1], [2]). In particular, accidents will still occur whenever autonomous vehicles and human drivers coexist. For these future scenarios, a sufficient amount of real world statistical data on traffic systems with different proportions of autonomous vehicles is not yet available. To overcome this lack of information, we propose a stochastic model that generates artificial data on both traffic flow and accidents. In this setting, we study the tradeoff between safety and efficiency as a function of the driving style of the individual vehicles. In the absence of real data, the sound design of future traffic systems requires such a strategy. Simulations that generate artificial data are a prerequisite for the anticipation of both future capabilities and risks associated with autonomous vehicles and their algorithms, advanced driver-assistance systems, and human drivers.

Vehicles in traffic systems are constantly in conflict with each other; autonomous vehicles and human drivers have to observe their environment, predict its future behavior and react accordingly in order to avoid accidents. Thereby, they control the distance to preceding vehicles, or – when turning or overtaking – they give way to other vehicles in order to avoid collisions. This requires the extrapolation of trajectories of potentially conflicting vehicles, the estimation of the size of safety gaps and decisions about when to stop and to wait, and when to proceed. These issues jointly appear at intersections turning them into a particularly risky location in traffic systems; Dresner and Stone [3] state that "vehicle collisions at intersections account for anywhere between 25% and 45% of all collisions".

In this paper, we focus on the traffic dynamics at intersections. We propose a model and present case studies for unsignalized intersections and discuss possible extensions for signalized ones. Intersections are modeled as multiple intersecting one-lane roads. On each of these roads, the basic movement of the vehicles is described by a microscopic car-following model. Cars need to control their distance to other vehicles in order to avoid rear-end collisions. At an intersection, additional conflicts between turning vehicles arise. We implement a conflict detection, fix a priority regime (right has right-of-way) and assume that vehicles will wait for emerging gaps if they have to give way. In reality, the three components – car-following, conflict detection, and conflict reaction – may be subject to errors of human drivers, possibly assisted by suitable technology. We model perceptional errors by stochastic processes which randomly fluctuate around the correct quantities.

The stylized model provides a conceptual framework for understanding the causal relationship between perceptional errors, (parametrized) driving style, accidents, and traffic flow. We study these features for different penetration rates of error-free autonomous vehicles in the traffic system. Our model captures the occurrence of two possible collision types: rear-end collisions resulting from low headways and frontal crashes in the context of turning maneuvers.

We provide a methodological basis and explain how traffic at intersections can be modeled by a system of coupled

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random ordinary differential equations.

This paper extends previous work [4] in multiple directions:

- Berkhahn et al. [4] primarily focuses on the Intelligent Driver Model with random misperception in the context of one-lane roads and heuristically discusses extensions to t-junctions. Rear-end collisions and collisions at t-junctions were analyzed separately. Now, we present a general and rigorous framework comprising both cases.
- This paper provides a comprehensive methodological analysis of a general class of random differential equations modeling both conflict detection and potential misperception. We use a state-of-the-art simulation technique and explain necessary adjustments in the context of the suggested model.
- In numerical case studies, we analyze the tradeoff between safety and efficiency.
- Besides scenarios with homogeneous traffic participants, we also analyze the heterogeneous case in which human drivers and autonomous vehicles coexist.

The paper is organized as follows: Section II reviews related literature, Section III introduces our model of intersections, Section IV explains the simulation methodology which we apply in various case studies. Simulation results are presented and discussed in Section V. Section VI concludes.

II. Literature Review

Literature on traffic modeling is vast. Our approach is based on a stochastic extension of microscopic traffic models which describe the movement of each vehicle individually. In particular, we adapt the Intelligent Driver Model (IDM), originally proposed by [5]. It belongs to the class of car-following models (also called follow-the-leader models). Random misperception may also be implemented in other car-following models, e.g., the Optimal Velocity Model (cf. [6] and [7]).

Several papers develop stochastic extensions of car-following models: Random fluctuations to the Optimal Velocity Model are implemented in [8]; [9] proposes and analyzes a stochastic "desired acceleration model"; [10] and [11] include stochastic processes within IDM. All these studies use randomness to explain naturally occurring fluctuations in traffic flow. In contrast, this paper focuses on stochastic processes that model perceptional errors which might trigger accidents, and it thereby rigorously extends our preliminary analysis in [4]. In the context of (deterministic) emergency braking scenarios, accidents are also analyzed in [12]; similar ideas are discussed in [13].

The stochastic character of perception and other cognitive processes of drivers is studied in [14]; Tversky and Kahneman's prospect theory is used as a framework for decision making in the face of risk. Closely related to the present paper is [15] where perception errors of autonomous vehicles are studied. Based on real data of [16], errors are calibrated using methods from time series analysis. The calibrated error models are incorporated into a commercial traffic simulation software, and the effects of errors are studied in test cases. The approach in [15] is complementary to ours. While we study the impact of errors on the number of accidents and traffic efficiency on an aggregate level, [15] does not capture the global impact of errors via variables such as traffic flow or the number of accidents, but focuses on its microscopic implications. Also, presumably due to substantial computational costs, the authors do not provide a statistical analysis of the consequences of the implemented errors – only four test trajectories in a braking scenario are presented, where an autonomous vehicle approaches a pedestrian. While [15] constructs a model that reflects many details of the collision dynamics of individual vehicles, our parsimonious model is sufficiently simple to study implications on the level of the whole traffic system.

The preceding papers mainly focus on unidirectional traffic, modeling vehicles on one-lane roads without intersections. Other models were developed for conflicting streams of traffic, e.g., unsignalized intersections or overtaking. Early contributions developed the idea of a *gap-acceptance function*: a certain time is needed to perform a potentially conflicting maneuver, defining a critical gap. If two successive conflicting vehicles exceed this critical gap, the maneuver is accepted, otherwise rejected. Gap-acceptance functions and assumptions on arrival times of vehicles at an intersection admit an analysis of delay times and capacities (c.f., e.g., [17] or [18]). Typically, heterogeneous human drivers obey different critical gaps. To reflect this issue, probability distributions are estimated from empirical data (e.g., [19], [20]). Various refinements have been suggested; for example, [21] includes risk assessments when entering an intersection; impatience is reflected by increasing the risk tolerance, the longer a vehicle waits.

Besides queuing theoretic approaches, some microscopic traffic models for intersections include the concept of gap-acceptance. A model based on cellular automata is suggested in [22]: a vehicle enters the intersection, if and only if enough cells of the conflicting stream of traffic are vacant. In the open source project SUMO, intersections are realized by comparing time slots in which potentially conflicting vehicles occupy the intersection (cf. [23]). An application of the gap-acceptance paradigm for lane-changing maneuvers was developed in [24]. In comparison, our model continuously detects potentially conflicting vehicles via trajectory extrapolation; potential conflicts trigger adjustments of the velocity of vehicles.

In the context of autonomous vehicles, models have been developed to demonstrate how traffic efficiency can be increased due to novel communication technologies. The benefits of inter-vehicle communication or communication with a central controller are studied in the context of *autonomous intersection management*, cf. [3]. The paper discusses incident mitigation techniques, but does not incorporate the possibility of endogenously occurring accidents. Auction and reservation based strategies for intersection management are, e.g., analyzed in [25]. Sophisticated intersection management is not considered in this paper; a comprehensive analysis of autonomous intersection managements in the face of risk and uncertainty would be an interesting topic for future research.

III. The Traffic Model

In this paper, we model traffic on intersecting lanes and incorporate the possibility of accidents caused by perceptional

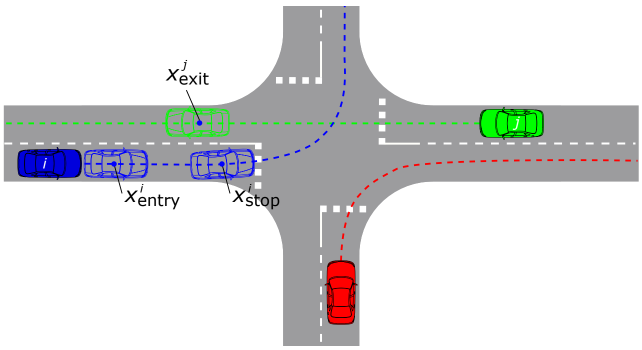


Fig. 1. Left-turning at an intersection.

errors. We assume that all vehicles move on prespecified paths, attempting to reach a target velocity. A vehicle accelerates, if its velocity is too low, unless conflicts with other vehicles are detected. We model two types of conflicts, namely an insufficient distance to the directly preceding vehicle, and vehicles crossing at intersections. In order to avoid collisions vehicles decelerate; the exact procedure is described below. Our model of an uncontrolled four-way intersection is illustrated in Fig. 1.

We first explain how we model a priority regime that mimics existing traffic regulations. Second, we describe a car-following model governing the movement on individual lanes. Third, we present a methodology for conflict detection. Fourth, we explain how vehicles adjust their speed. Our model incorporates errors due to *random misperception*. Estimates of distances and velocities are input quantities to the car-following model; conflict detection and reactions of vehicles depend on these variables. Our model assumes that the estimates of human drivers are subject to randomly fluctuating errors that are captured by suitable stochastic processes.

We begin with formal notation. The set $\mathcal{M} = \{1, 2, 3, \dots\}$ consists of all considered vehicles. We associate each vehicle $i \in \mathcal{M}$ with three stochastic processes with continuous paths, denoted by $(\varepsilon_t^{i,1})_{t\geq 0}, (\varepsilon_t^{i,2})_{t\geq 0}, (\varepsilon_t^{i,3})_{t\geq 0}$ that fluctuate around the value 1. The processes are multipliers that distort the true values of velocities and distances and thereby capture random misperception. Throughout the paper, for each vehicle i, the first process $(\varepsilon_t^{i,1})$ refers to the misperception of vehicle i's own velocity; the second process $(\varepsilon_t^{i,2})$ models errors in the estimation of the velocity of other vehicles; the third process $(\varepsilon_t^{i,3})$ captures estimation errors of relevant distances. Further assumptions on the structure of these processes will be described in Section V.

A. Priority Regime

The dynamics of uncontrolled intersections mimics German traffic regulations; of course, the approach could be amended to capture other countries. In Germany, "vehicles coming from the right have the right of way" unless specified otherwise.

While often applicable, this simple rule does not always produce a solution: If vehicles come from all directions at the same time, traffic may be deadlocked. In these situations, the following additional traffic rule applies [26, Section 11 Special traffic situations, (3)]:

> Moreover, anyone who, according to traffic rules, may proceed or otherwise has the right of way must relinquish this priority if the traffic situation so requires; a person not having the right of way may proceed only if the person having the right of way has signaled to them to do so.

In our model, we check if there is a cycle in the chain of priority. If this is the case, all waiting vehicles i observe independent exponentially distributed waiting times $t^i_{\text{solve}} \sim \text{Exp}(\lambda),\ \lambda > 0$, with expectation $\mathbb{E}(t^i_{\text{solve}}) = \lambda^{-1}$; the vehicle whose clock rings first will give up its priority.

Remark 1. *Of course, the behavior stipulated by traffic regulations is not efficient. For autonomous vehicles, one could envision control algorithms that lead to both safer and more efficient outcomes. Research on this topic runs under the keyword* autonomous intersection management *(cf. also Section II).*

B. Car-Following Model

The paths of vehicles, also called trajectories, lie on one-dimensional curves that describe the geometry of the traffic system; this is illustrated in Fig. 1. In our model, the paths of vehicles are prespecified and fixed; speed can be adjusted. Vehicles with the same trajectory follow each other. Their behavior is modeled by the *Intelligent Driver Model with Random Misperception* (IDMrm) as developed by our research group in [4]. IDMrm is a stochastic extension of the classical IDM with a bound on maximal deceleration in which perceptional errors are incorporated. On each of the one-dimensional curves that capture potential paths of vehicles, we fix an origin. For any vehicle i that moves along this curve we denote by $x^i(t)$ the distance of the vehicle's position at time t to the origin along the section of the curve, i.e., the arc length of the corresponding segment of its trajectory; the time derivative $v^i(t)$ of $x^i(t)$ is the velocity of vehicle i at time t.

Vehicles are controlled on the basis of measurements of distances and velocities. But these measurements are subject to errors. Distortions are captured by multiplicative factors $(\varepsilon_t^{i,1}), (\varepsilon_t^{i,2})$, and $(\varepsilon_t^{i,3})$. On its one-dimensional trajectory, each vehicle computes its acceleration based on the perceived values of its own velocity $\varepsilon_t^{i,1} v^i(t)$, its perceived distance to the preceding vehicle $\Delta_{\text{per}} x^i(t)$ and its perceived approaching rate $\Delta_{\text{per}} v^i(t)$. The identity of the preceding vehicle may change, since vehicles can turn at the intersection, and we denote this vehicle by $i_{\text{pre}}(t)$. Letting $\Delta x^i(t)$ be the exact distance to the preceding vehicle along the path, we may formally define the perceived quantities:

$$\Delta_{\text{per}} v^i(t) := \varepsilon_t^{i,1} v^i(t) - \varepsilon_t^{i,2} v^{i_{\text{pre}}(t)}(t), \tag{1}$$
$$\Delta_{\text{per}} x^i(t) := \varepsilon_t^{i,3} \Delta x^i(t). \tag{2}$$

For a vehicle i on a one-dimensional line its acceleration is computed as

$$a^i_{\mathrm{IDMrm}}(t) := a^i_{\max}\left(1-\left(\frac{\varepsilon^{i,1}_t v^i(t)}{v^i_{\mathrm{d}}}\right)^{\delta}\right.$$
$$\left. -\left(\frac{s^*(\varepsilon^{i,1}_t v^i(t), \Delta_{\mathrm{per}} v^i(t))}{\Delta_{\mathrm{per}} x^i(t)}\right)^2\right)$$

where $s^*(s_1, s_2) = s_0 + s_1 T + \frac{s_1 s_2}{2\sqrt{a^i_{\max} b}}$; we set $s^*(\varepsilon^{i,1}_t v^i(t), \Delta_{\mathrm{per}} v^i(t)) \cdot (\Delta_{\mathrm{per}} x^i(t))^{-1} := 0$ if there is no preceding vehicle. The quantity $a^i_{\max} > 0$ is the maximal acceleration of the i-th vehicle, and $v^i_{\mathrm{d}} > 0$ denotes its desired velocity. The other parameters originate from the classic IDM model, and we refer to [5] for a detailed explanation.

Remark 2. *In Eq.* (1) *&* (2)*, misperception of human drivers is modeled by multiplicative errors. Multiplicative errors are relative errors, and their advantage is that their size scales with the magnitude of the true values. Alternatively, additive errors could be chosen. The simulation method described in Section IV could easily be adapted.*

C. Conflict Detection at Intersections

The control of individual vehicles and traffic flow depends on priority regimes. For each vehicle i, we denote by $\mathcal{M}^i_{\mathrm{rel}}(t) \subseteq \mathcal{M}$ the family of vehicles to which it has to give way. These are vehicles approaching the intersection which are coming from the right; this includes oncoming vehicles when vehicle i is turning left.

Vehicles always stay on their prespecified paths, signaling their turning intentions correctly. At time t, trajectories of other vehicles are extrapolated into the future for a fixed time horizon of length t^* on the basis of potentially distorted estimates of distances and velocities (see [27] for more details on trajectory extrapolation). Using the extrapolated trajectories, one computes an estimate of vehicle i's distance to another vehicle j at future time u which is denoted by $\hat{d}^{ij}(u)$; vehicles' paths may be located on different one-dimensional curves, and for this reason we measure $\hat{d}^{ij}(u)$ as the usual Euclidean distance in the two-dimensional plane into which the trajectories are embedded. Note that $\hat{d}^{ij}(u)$ implicitly depends on t, but we suppress this dependence in the notation, since it will always be clear from the context. As in the context of car-following, we assume that distances to other vehicles are misperceived. Analogously, we assume that vehicle i perceives its own distance to vehicle j as $\varepsilon^{i,3}_t \hat{d}^{ij}(u)$, i.e., the estimate $\hat{d}^{ij}(u)$ is distorted by the multiplier $\varepsilon^{i,3}_t$.

If $j \in \mathcal{M}^i_{\mathrm{rel}}(t)$, vehicle i detects a conflict at time t, if $\varepsilon^{i,3}_t \hat{d}^{ij}(u) < d_{\mathbf{s}}$ for a safety threshold $d_{\mathbf{s}} \geq 0$ and $t \leq u \leq t+t^*$, i.e., the (distorted) extrapolated distance between the two vehicles i and j falls below the safety threshold at a future time horizon. In addition, if vehicle i is in the area of the intersection and detects a conflict with another vehicle j that has the right of way, vehicle i keeps the conflict in mind until vehicle j leaves the area of the intersection. In order to make this precise, we introduce fixed locations x^i_{stop}, x^i_{entry}, and x^j_{exit}:

- x^i_{stop} is the position on i's trajectory where vehicle i should stop in order to let conflicting vehicles j pass;
- x^i_{entry} is the position such that vehicle i, driving with desired speed v^i_d, is able to come to a complete stop at x^i_{stop} using its maximal deceleration;
- vehicle j has passed the intersection if it has reached x^j_{exit};
- we say i is in the area of the intersection at time t if $x^i_{\mathrm{entry}} < x^i(t) < x^i_{\mathrm{exit}}$.

In summary, we define the set $\mathcal{M}^i_{\mathrm{conflict}}(t)$ of conflicting vehicles by

$$\begin{aligned}&\mathcal{M}^i_{\mathrm{conflict}}(t)\\ &:= \left\{ j \in \mathcal{M}^i_{\mathrm{rel}}(t) \mid \exists\, u \in [t, t+t^*] \colon \varepsilon^{i,3}_t \hat{d}^{ij}(u) < d_{\mathbf{s}} \right\}\\ &\cup \left(\bigcup_{u<t} \{ j \in \mathcal{M}^i_{\mathrm{conflict}}(u) | x^i_{\mathrm{entry}} < x^i(u), x^j(t) < x^j_{\mathrm{exit}} \} \right).\end{aligned}$$

D. Conflict Reaction

If the set of conflicting vehicles $\mathcal{M}^i_{\mathrm{conflict}}(t)$ is nonempty, vehicle i reacts to this situation. We distinguish two cases: stopping, or decelerating when stopping is unnecessary.

- Complete stop: If vehicle i is in position $x^i(t)$ with velocity $v^i(t)$, the constant (negative) acceleration to stop at x^i_{stop} equals
$$a^i_{\mathrm{stop}}(t) := -\frac{(v^i(t))^2}{2(x^i_{\mathrm{stop}} - x^i(t))}.$$
The duration of this maneuver is $t^i_{\mathrm{stop}}(t) := -v^i(t)/a^i_{\mathrm{stop}}(t)$.
- Deceleration: Stopping is not always necessary. Consider a vehicle i and a conflicting vehicle j. We assume that vehicle i bases its acceleration on a simplified prediction of vehicle j by assuming that j's velocity is fixed. The time it would take for vehicle j to leave the intersection with fixed speed $v^j(t)$ is $t^j_{\mathrm{exit}}(t) := (x^j_{\mathrm{exit}} - x^j(t))/v^j(t)$. If $t^i_{\mathrm{stop}}(t) > t^j_{\mathrm{exit}}(t)$, vehicle i does not intend to stop, but only to slow down. The constant deceleration such that vehicle i arrives at x^i_{stop} at the predicted time equals
$$a^{ij}_{\mathrm{break}}(t) := \left(\frac{x^i_{\mathrm{stop}} - x^i(t)}{t^j_{\mathrm{exit}}(t)} - v^i(t)\right)\frac{2}{t^j_{\mathrm{exit}}(t)}.$$

Conflict reaction to vehicles $j \in \mathcal{M}^i_{\mathrm{conflict}}(t)$ is modeled by bounding the acceleration from above by

$$a^{ij}_{\mathrm{conflict}}(t) := \begin{cases} a^{ij}_{\mathrm{break}}(t), & \text{if } t^i_{\mathrm{stop}}(t) > t^j_{\mathrm{exit}}(t),\\ a^i_{\mathrm{stop}}(t), & \text{if } t^i_{\mathrm{stop}}(t) \leq t^j_{\mathrm{exit}}(t).\end{cases}$$

We do, however, not assume that these quantities depend on the correct distances or velocities, but on their perceived values and replace the arguments of the functions accordingly, i.e., $v^i(t)$ by $\varepsilon^{i,1}_t v^i(t)$, $x^i_{\mathrm{stop}} - x^i(t)$ by $\varepsilon^{i,3}_t(x^i_{\mathrm{stop}} - x^i(t))$, $x^j_{\mathrm{exit}} - x^j(t)$ by $\varepsilon^{i,3}_t(x^j_{\mathrm{exit}} - x^j(t)$, and $v^j(t)$ by $\varepsilon^{i,2}_t v^j(t)$, $j \in \mathcal{M}\backslash\{i\}$.

E. Intersection Dynamics Subject to Random Misperception

The motion of the vehicles can be expressed as a system of coupled random ordinary differential equations:

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t} x^i(t) = \max\{v^i(t), 0\}, \\ \frac{\mathrm{d}}{\mathrm{d}t} v^i(t) = \max\Big\{a^i_{\min}, \min\Big\{a^i_{\mathrm{IDMrm}}(t), \\ \qquad\qquad \min_{j\in\mathcal{M}^i_{\mathrm{conflict}}(t)} a^{ij}_{\mathrm{conflict}}(t)\Big\}\Big\} \\ x^i(t^i_0) = 0,\ v^i(t^i_0) = v^i_0,\ t \geq t^i_0,\ i \in \mathcal{M}. \end{cases} \tag{3}$$

Velocities are bounded from below by 0. The minimal acceleration of vehicle i is set to $a^i_{\min}$; this is both realistic and necessary, if accidents are admissible. The acceleration of a vehicle i is the minimum of $a^i_{\mathrm{IDMrm}}(t)$ (to control the distance to the preceding vehicle) and $\min_{j\in\mathcal{M}^i_{\mathrm{conflict}}(t)} a^{ij}_{\mathrm{conflict}}(t)$ (to solve all conflicts in the intersection simultaneously). Each vehicle i enters the system on its path with an initial velocity $v^i_0 \geq 0$ at time t^i_0 and is removed from the system once it reaches the end of its path.

Remark 3. *The described conflict reaction is closely related to microscopic gap-acceptance models. Instead of critical gaps in time, we measure the distance to conflicting vehicles and adjust the velocity accordingly. Gap-acceptance models reflect heterogeneity via probability distributions. A similar approach could be applied to the safety threshold in our model and the adjustment of the velocities.*

Remark 4. *In this paper, we study uncontrolled bi-directional two-lane intersections. Our approach can be generalized to other priority regimes:*

- *Prioritized roads can be modeled by adjusting $\mathcal{M}^i_{\mathrm{rel}}(t)$, i.e., the set of vehicles to which one needs to give way.*
- *To model a signalized intersection, one could include another time-dependent acceleration term $a^i_{\mathrm{signal}}(t)$ forcing vehicle i to decelerate on red. The resulting acceleration would be $\min\{a^i_{\mathrm{IDMrm}}(t), a^i_{\mathrm{signal}}(t), \min_{j\in\mathcal{M}^i_{\mathrm{conflict}}(t)} a^{ij}_{\mathrm{conflict}}(t)\}$. Also, other relevant conflicts and other types of misperception could be integrated in a more comprehensive model.*

Remark 5. *Mathematically, the model in* (3) *is a system of coupled random ordinary differential equations. Random ordinary differential equations (RODEs) are ordinary differential equations whose right-hand side depends on some stochastic process. Pathwise, these are non-autonomous classical ordinary differential equations and can be solved by deterministic calculus. Local existence of a weak solution is guaranteed by Theorem 1 (cf. Appendix A), i.e., we find trajectories $x^i(t)$ which satisfy* (3) *for Lebesgue almost all times. However, many classical numerical methods are inappropriate due to the roughness of the paths of the stochastic processes. Suitable schemes will be explained in the next section.*

Computing $a^{ij}_{\mathrm{conflict}}(t)$ is expensive. This effort can be reduced by virtue of the following simple lemma: If vehicle i anyway intends to stop due to some conflicting vehicle j, it does not need to analyze other conflicting vehicles anymore. The proof is trivial.

Lemma 1. *Let $i \in \mathcal{M}$ and $j \in \mathcal{M}^i_{\mathrm{conflict}}(t)$. The following statements hold:*

1) $a^i_{\mathrm{stop}}(t) \leq a^{ij}_{\mathrm{break}}(t)$,
2) *If there exists $j^* \in \mathcal{M}^i_{\mathrm{conflict}}(t)$ such that $a^{ij^*}_{\mathrm{conflict}}(t) = a^i_{\mathrm{stop}}(t)$, then $\min_{j\in\mathcal{M}^i_{\mathrm{conflict}}(t)} a^{ij}_{\mathrm{conflict}}(t) = a^i_{\mathrm{stop}}(t)$.*

Random misperception may trigger accidents, i.e., collisions of vehicles. In order to capture this, we denote by $A^i(t)$ the area that is occupied by vehicle $i \in \mathcal{M}$ at time t; this is modeled by an ellipse in the two-dimensional plane. A *collision* occurs if $A^i(t) \cap A^j(t) \neq \emptyset$ for $i, j \in \mathcal{M}$ and $t \geq 0$. In this case, we set the velocity of the vehicles i and j to 0 and adjust the dynamics of the traffic system (3) accordingly. Moreover, we trigger an exponentially distributed waiting time

$$t_{\mathrm{removal}} \sim \mathrm{Exp}(\gamma), \quad \gamma > 0$$

with expectation $\mathbb{E}(t_{\mathrm{removal}}) = 1/\gamma$. Meanwhile, other vehicles may crash into the existing collision; however, after t_{removal} has passed, *all* vehicles that are involved in this particular accident are removed from the model. Of course, simultaneously other accidents may occur at other places.

F. Calibration

The model includes three dimensions: car-following, conflict detection and reaction, and misperception. The aim of the model is to provide an experimental lab that allows to envision future traffic systems. In particular, future advanced driver-assistance systems will modify the behavior of human drivers, and we will also study the coexistence of human drivers and autonomous vehicles. This implies that the model cannot fully be calibrated to statistical data. In fact, the purpose of the model is to generate artificial data of novel traffic systems that do not yet exist. However, calibration needs to be discussed in the context of suitable benchmarks. Our model will deviate from these benchmarks, and it can be compared to them.

- **Car-following models** can be calibrated from either aggregate or disaggregate traffic data (see, e.g., [5], [28], [29], [30], [31]). These may also serve as benchmark models of autonomous vehicles; however, their behavior will deviate from current traffic data due to the increased capabilities of the vehicles and their flexible and partially unknown future design.
- **Conflict detection and reaction** is captured by a stylized model in this paper. A benchmark can be estimated on the basis of similar methodologies previously suggested for gap-acceptance models (see, e.g., [19], [20], [21]).
- **Perception errors** of human drivers could be estimated on the basis of statistical data (see, e.g., [32], [33]). In the future, the size of these errors may be further reduced by improved technology, e.g., advanced driver-assistance systems.

IV. Simulation Method

On a pathwise level, RODEs are non-autonomous ordinary differential equations; classical first order methods from deterministic calculus can be applied to solve them. RODEs depend, however, on stochastic processes which typically possess paths of unbounded variation that are nowhere differentiable. Typical examples are (fractional) Brownian motion and related processes such as the Ornstein-Uhlenbeck process that we will consider in this paper. Due to their insufficient smoothness, many classical numerical methods are not appropriate; the reason is that standard arguments for the error analysis of numerical schemes are not applicable anymore, since these are often based on Taylor expansions requiring sufficient regularity (we refer to [34] for a more detailed discussion of this issue).

These challenges are addressed by simulation schemes that are specifically taylored for RODEs. To approximate the solutions, we employ the γ-RODE-Taylor scheme (cf. [35]). This method requires that there exists $\theta^i = (\theta^{i,1}, \theta^{i,2}, \theta^{i,3})^\top \in (0,1]^3$ such that each component process $(\varepsilon_t^{i,k})_{t\geq 0}$ is Hölder continuous for all exponents $\eta^{i,k}$ satisfying $0 < \eta^{i,k} < \theta^{i,k},\ k = 1,2,3$ (cf. Assumption 3.1 in [35]).

We consider a time discretization $\mathbb{T} = \{t_0, t_1, ...\}$; for the individual time points and for all $i \in \mathcal{M}$ we determine an approximate solution $\begin{pmatrix} x_k^i \\ v_k^i \end{pmatrix}_{k=0,1,2,...}$ to our system (3).

Consider the iteration interval $[t_k, t_{k+1}]$. In order to compute for a fixed $i \in \mathcal{M}$ the update $\begin{pmatrix} x_{k+1}^i \\ v_{k+1}^i \end{pmatrix}$ we treat $\begin{pmatrix} x_k^j \\ v_k^j \end{pmatrix}$ for $j \neq i$ as fixed exogenous input values. We compute $\mathcal{M}^i_{\text{conflict}}(t_k)$ and $i_{\text{pre}}(t_k)$, and fix these values for the iteration interval $[t_k, t_{k+1}]$. Under these assumptions, the right-hand side of the evolution equation for i, as given in (3), can be rewritten in terms of a function $f\colon \mathbb{R}^3 \times \mathbb{R}^2 \to \mathbb{R}^2$ with arguments $(\varepsilon_t^{i,1}, \varepsilon_t^{i,2}, \varepsilon_t^{i,3})^\top \in \mathbb{R}^3$ and $(x^i(t), v^i(t))^\top \in \mathbb{R}^2$. We replace f by a suitable infinitely differentiable approximation that we again denote by f.

In the case studies in the next section, we consider error processes $(\varepsilon_t^{i,1})_{t\geq 0}, (\varepsilon_t^{i,2})_{t\geq 0}, (\varepsilon_t^{i,3})_{t\geq 0}$ with $\theta^i = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})^\top$ (cf. Subsection V-B). Setting $\gamma = 1$, we obtain the pathwise γ-RODE-Taylor scheme

$$\begin{aligned}\Phi_1(z,t,h) &:= z \;+\; h \cdot f(\varepsilon_t^i, z) \\ &\quad +\; \frac{h}{n}\sum_{k=1}^{3} \partial_{w_k} f(\varepsilon_t^i, z) \sum_{j=1}^{n-1} \Delta\varepsilon_{t,\tau_j}^{i,k},\end{aligned}$$

where $\tau_j = t + \frac{j}{n}\cdot h$, $\Delta\varepsilon_{t,\tau_j}^{i,k} = \varepsilon_{\tau_j}^{i,k} - \varepsilon_t^{i,k}$ and $n = \left\lceil h^{1-\frac{2}{1-2\xi}} \right\rceil$ for $\xi > 0$ small. Here, $\lceil\cdot\rceil$ is a Gauss-bracket, and $\partial_{w_1}, \ldots, \partial_{w_3}$ denote the partial derivatives with respect to the three error components of f. The derivatives of f are approximated by difference quotients. The stepwise order of convergence equals $\gamma = 1$. The approximation of the solution of (3) for vehicle $i \in \mathcal{M}$ at time t_{k+1} is given by

$$\begin{pmatrix} x_{k+1}^i \\ v_{k+1}^i \end{pmatrix} = \Phi_1\left(\begin{pmatrix} x_k^i \\ v_k^i \end{pmatrix}, t_k, \Delta t_{k+1}\right)$$

with $\Delta t_k := t_k - t_{k-1}$.

V. Case Study

A. Performance Measures

We evaluate our model in terms of risk and efficiency. We study

- the *number of accidents*, the *number of collided vehicles*, and the *number of collided vehicles per accident* as quantitative measures of the riskiness of the system, and
- *network traffic flow* as a measure of system efficiency.

The length of the simulation period is denoted by T_{sim}.

1) Network Traffic Flow: We assign to each vehicle $i \in \mathcal{M}$ a final destination dest_i on its path. Network traffic flow Q is measured by the number of vehicles $i \in \mathcal{M}$ per time unit that reach their destinations:

$$Q = \frac{\text{card}\{j \in \mathcal{M}\colon \exists\ t \leq T_{\text{sim}}\colon x^j(t) = \text{dest}_j\}}{T_{\text{sim}}},$$

where card denotes the cardinality. The corresponding sample mean is denoted by $\hat{Q}$.

2) Number of Accidents: A first proxy for the safety of traffic systems is given by the number of accidents per time unit:

$$\begin{aligned} f_{\text{acc}} = T_{\text{sim}}^{-1} \cdot \text{card}\{&\emptyset \neq M \subset \mathcal{M}\colon \\ &\exists\ t \leq T_{\text{sim}}\ \forall\ i \in M\colon A^i(t) \cap A^{M\setminus\{i\}}(t) \neq \emptyset \\ &\text{and}\ \forall\ t \leq T_{\text{sim}}\colon A^M(t) \cap A^{M^c}(t) = \emptyset\}\end{aligned}$$

where $M^c := \mathcal{M} \setminus M$ and $A^M(t) := \bigcup_{i\in M} A^i(t)$. The corresponding sample mean is denoted by $\hat{f}_{\text{acc}}$. An accident is the event that multiple cars are jointly involved in collisions. A collision occurs, if the associated areas of two vehicles intersect.

3) Number of Collided Vehicles: The number of vehicles per time unit that are involved in accidents is given by

$$\begin{aligned} f_{\text{veh}} = T_{\text{sim}}^{-1} \cdot \text{card}\{i \in \mathcal{M}\colon \exists\ t \leq T_{\text{sim}}\ \exists\ j \in \mathcal{M} \setminus \{i\}\colon \\ A^i(t) \cap A^j(t) \neq \emptyset\}.\end{aligned}$$

The corresponding sample mean is denoted by $\hat{f}_{\text{veh}}$.

4) Number of Collided Vehicles per Accident: A measure for the average severity of an accident is the number of collided vehicles divided by the number of accidents:

$$g_{\text{veh/acc}} = \frac{f_{\text{veh}}}{f_{\text{acc}}}.$$

Its sample mean is denoted by $\hat{g}_{\text{veh/acc}}$.

B. Misperception Model

Perception of the environment is subject to errors. In our multiplicative error model, mean-reverting processes are capable of capturing noisy deviations; mean-reverting processes are stochastic processes that randomly fluctuate around fixed values. In contrast, permanent malfunctions can, for example, be modeled by jump processes such as continuous time Markov chains. Both aspects may also be combined in a joint model.

In this paper, we focus only on the first dimension of misperception, i.e., random noise that distorts perceived quantities around their true values. An important example of a mean-reverting stochastic process that fluctuates around a constant level is the Ornstein-Uhlenbeck process. This process was also used in [4] to model perceptional errors. We refer to [15] for potentially more realistic, but less tractable alternative approaches.

Definition 1 (Ornstein-Uhlenbeck Process)**.** *Let* $\beta \in \mathbb{R}$ *and* $\alpha, \sigma > 0$*. A stochastic process* $(\varepsilon_t)_{t\geq 0}$ *is called an* Ornstein-Uhlenbeck process*, if* $\varepsilon_0 = a \in \mathbb{R}$ *and* $(\varepsilon_t)_{t\geq 0}$ *solves the following stochastic differential equation:*

$$d\varepsilon_t = \alpha(\beta - \varepsilon_t)dt + \sigma dW_t,$$

where $(W_t)_{t\geq 0}$ *denotes a one-dimensional standard Brownian motion.*

Typical simulated paths are shown in Fig. 2. For more details regarding simulation and interpretation, we refer to [4] and [36].

In our case studies, we analyze two scenarios – a homogeneous and a heterogeneous scenario:

1) In the homogeneous case, we assume that all $(\varepsilon_t^{i,1}), (\varepsilon_t^{i,2})$ and $(\varepsilon_t^{i,3})$ are independent and identically distributed Ornstein-Uhlenbeck processes with parameters $a = \beta = \alpha = 1$ and varying values of the parameter σ which controls the volatility of the process.
2) In the heterogeneous case, we assume that vehicles with and without misperception coexist. For the latter, we assume $\varepsilon_t^{i,1} = \varepsilon_t^{i,2} = \varepsilon_t^{i,3} \equiv 1$, capturing perfect perception.

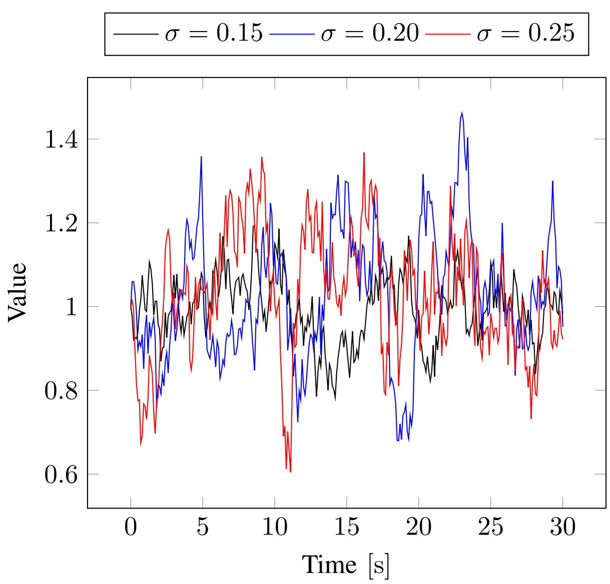


Fig. 2. Simulated paths of an Ornstein-Uhlenbeck process (ε_t) for different values of σ with $\alpha = 1$, $\varepsilon_0 = 1$, $\beta = 1$.

C. Scenario Description

The intersection consists of two two-lane roads of length $210\,\mathrm{m}$, each having a width of $10\,\mathrm{m}$. For this case study, we generate vehicles in the following way: The intersection can be approached from four directions, i.e., vehicles are generated at four origins. We create vehicles with an exponentially distributed headway such that the expected rate of vehicles per time unit at each source is $150\,\mathrm{veh}/\mathrm{h}$. Larger gaps may randomly emerge between vehicles – allowing vehicles to turn that wait at the intersection. If substantial traffic jams occur, our simulation delays the generation of new vehicles such that traffic flow cannot completely break down.

When a vehicle is generated, it chooses with probability $1/3$ one of the following paths: turn right, go straight, turn left. Its initial velocity is set to the velocity of the preceding vehicle. If there is no preceding vehicle, it starts with its desired velocity.

We simulate the traffic system for a duration of $T_{\text{sim}} = 600\,\mathrm{s}$. To reach a representative, potentially stationary state of the Markovian model, we implement a burn-in period of $100\,\mathrm{s}$. Data for the computation of the relevant statistics are recorded afterwards. The model is simulated on an equidistant time grid with $\Delta t_k \equiv 0.1\,\mathrm{s}$. We sample repeatedly and compute averages of Q, f_{acc}, and f_{veh} from the empirical distributions.

Remark 6. *A generation rate of* $150\,\mathrm{veh}/\mathrm{h}$ *per source corresponds to a scenario of low to moderate traffic volume. In the error-free case, the generation rate could be increased. Real-world traffic regulations require the installation of traffic signals if traffic volumes are slightly larger: According to German regulations (cf. [37]), a traffic light should be installed if total traffic flow exceeds* $800\,\mathrm{veh}/\mathrm{h}$*; US regulations (cf. [38]) specify that the sum of traffic flow on the major lanes should not exceed* $500\,\mathrm{veh}/\mathrm{h}$ *and the maximum of flow on the minor lanes should be below* $150\,\mathrm{veh}/\mathrm{h}$*. In our case study, all sources have the same generation rate of maximally* $150\,\mathrm{veh}/\mathrm{h}$ *without any distinction between major and minor lanes.*

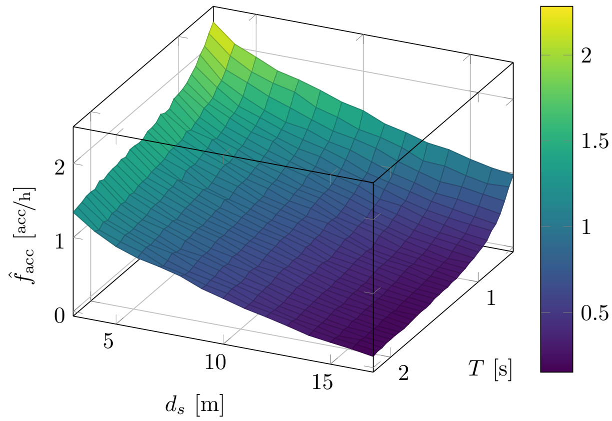
D. Simulation Results

We independently simulate the four-way intersection $20,000$ times. The parameters underlying the simulation are displayed in Appendix B. We separately analyze the homogeneous and the heterogeneous case and study the effect of two important control parameters:

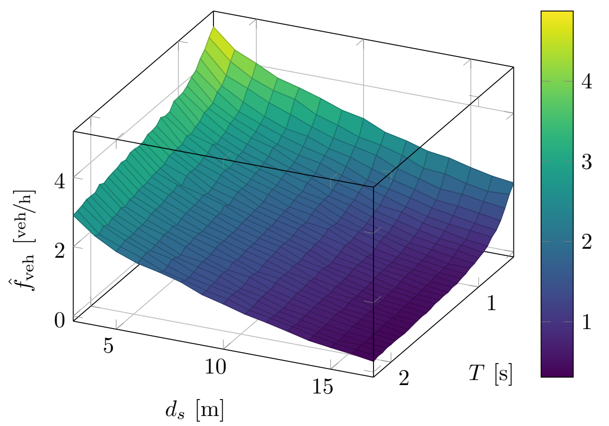
- We vary the time headway T of the vehicles (T is a parameter of the car-following model IDMrm), and
- the safety distance $d_{\mathbf{s}}$ which controls whether an oncoming vehicle is classified as conflicting or not.

1) Homogeneous Traffic: We assume that all vehicles are subject to misperception as described in Section V-B. This occurs if only human drivers are present. Fig. 3 displays the quantitative measures of the risk of the system: Fig. 3a shows the number of accidents, Fig. 3b the number of collided vehicles. Of course, safety increases when time headway and safety threshold are increased. Both graphs are quite similar in shape, since for all parameters accidents involve on average about 2 to 2.2 vehicles, see Fig. 3c. Due to low velocities ($10\,\mathrm{m}/\mathrm{s}$) and a moderate rate at which vehicles are generated, vehicles are able to react to most collisions.

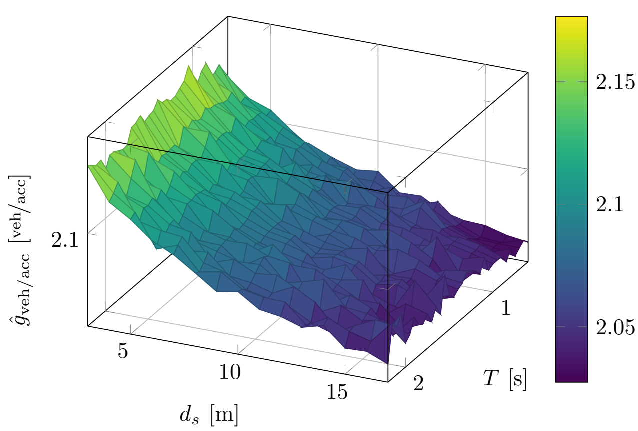
The efficiency of the traffic system is shown in Fig. 4: Fig. 4a shows a surface plot, Fig. 4b a top view of the same graph. The function is strictly concave and has a unique maximum. This captures the tradeoff between risk and efficiency: If $d_{\mathbf{s}}$ or T are small, the number of accidents is high and the intersection is blocked. Hence, traffic flow is low. With increasing $d_{\mathbf{s}}$ and T, the number of accidents decreases and traffic flow increases. However, traffic becomes inefficient, if $d_{\mathbf{s}}$ or T are large, i.e., if vehicles drive too carefully; in this case, traffic flow decreases. Accidents do still occur in the



(a) Number of Accidents

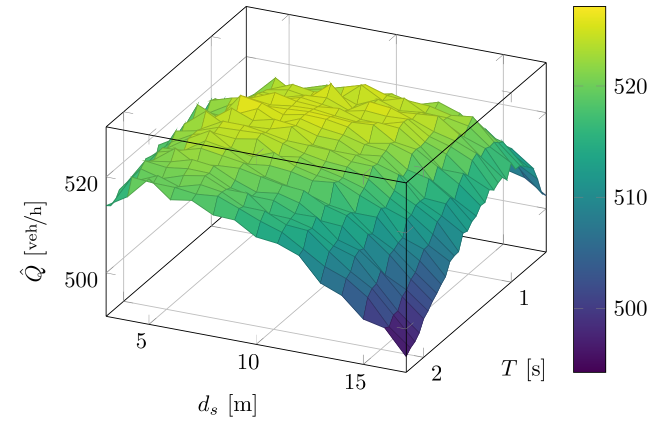


(b) Number of Collided Vehicles

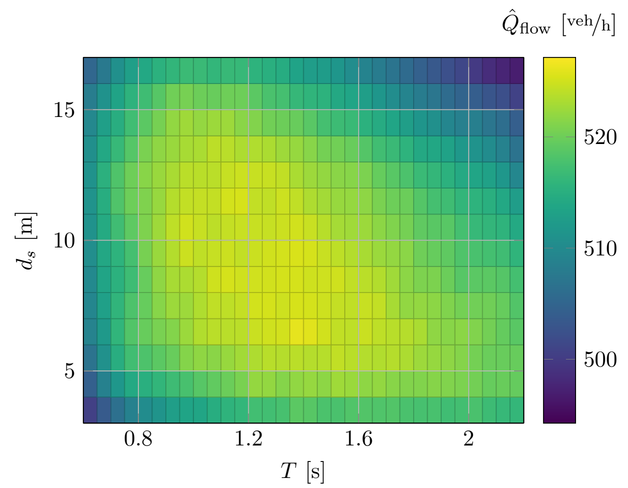


(c) Number of Collided Vehicles per Accident

Fig. 3. Number of accidents, number of collided vehicles and number of collided vehicles per accident for $\sigma = 0.2$ and varying d_s and T with $20,000$ independent simulations for each parameter combination.



(a) Surface Plot



(b) Top View

Fig. 4. Average traffic flow for $\sigma = 0.2$ and varying $d_{\mathbf{s}}$ and T with $20,000$ independent simulations for each parameter combination.

most efficient traffic flow scenario. The findings generalize our preliminary results in [4].

Our model admits two types of collisions: rear-end collisions and collisions at the intersection. Fig. 5 depicts the fraction of rear-end accidents among all accidents. Within the considered parameter range, T has a stronger influence, but also $d_{\mathbf{s}}$ has an impact by reducing the number of collisions at the intersections. The dependence of Q and f_{acc} on σ is shown in Fig. 6: Of course, with increasing volatility more accidents occur and traffic flow decreases. These effects are superlinear.

2) Heterogeneous Traffic: We now consider the coexistence of vehicles with perfect perfection and misperception, analyzing their impact on traffic efficiency and accidents. Perfect perception might be associated with autonomous vehicles, while random misperception occurs in human drivers, possibly using advanced driver-assistance systems that tame the size of errors. The parameter $\rho \in [0, 1]$ denotes the penetration rate of vehicles with perfect perception. Sampling independent Bernoulli-distributed random variables with parameter ρ, we randomly pick the type of each vehicle.

Fig. 7 shows the average traffic flow and number of acci-

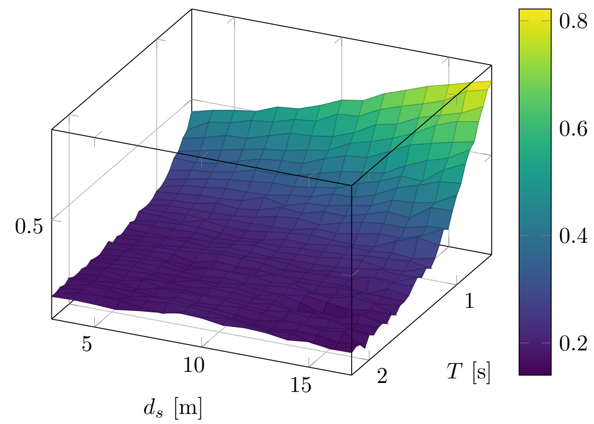


Fig. 5. Fraction of rear-end accidents for $\sigma = 0.2$ and varying $d_{\mathbf{s}}$ and T with $20,000$ independent simulations for each parameter combination.

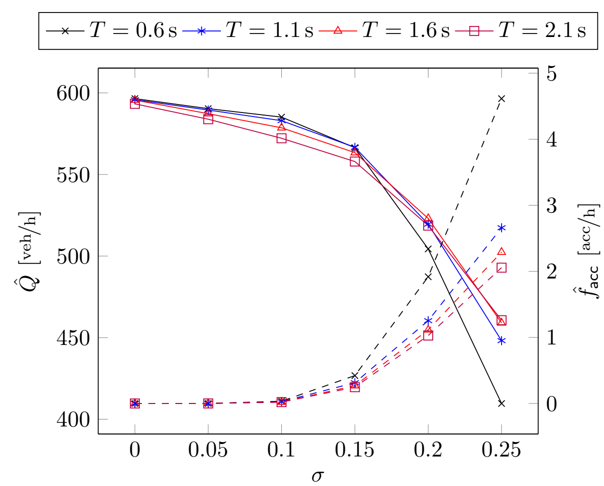
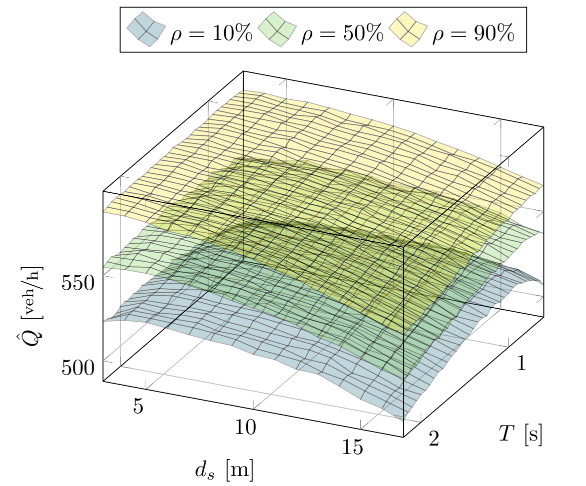
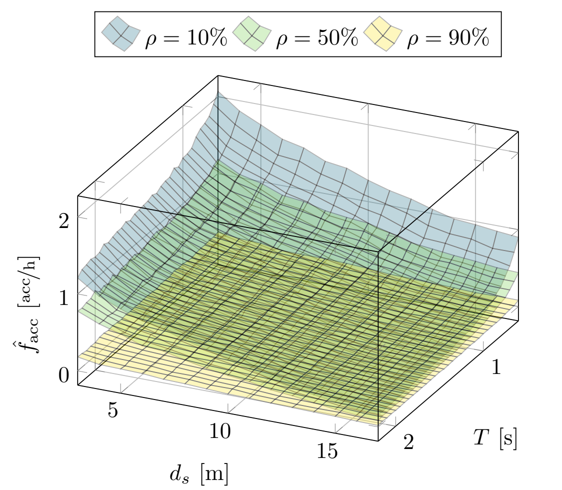


Fig. 6. Average network traffic flow and number of accidents for $d_{\mathbf{s}} = 5\,\mathrm{m}$, fixed T, and varying σ with $20,000$ independent simulations for each parameter combination. Dashed lines correspond to number of accidents, solid lines to flows.



(a) Traffic Flow



(b) Number of Accidents

Fig. 7. Average network traffic flow and number of accidents for different penetration rates ρ, $\sigma = 0.2$, and varying $d_{\mathbf{s}}$ and T with $20,000$ independent simulations for each parameter combination.

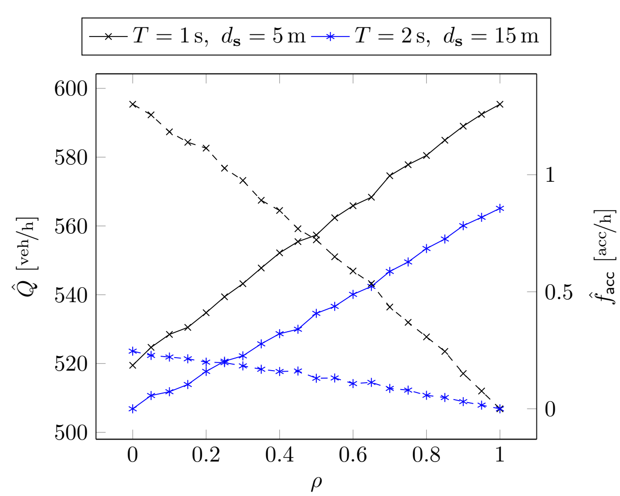


Fig. 8. Average network traffic flow and number of accidents for $\sigma = 0.2$, fixed T and $d_{\mathbf{s}}$, and varying ρ with $20,000$ independent simulations for each parameter combination. Dashed lines correspond to number of accidents, solid lines to flows.

dents. As in the homogeneous case, we vary the time headway T and the safety distance $d_{\mathbf{s}}$. Here, we consider penetration rates $\rho \in \{10\%,\ 50\%,\ 90\%\}$. Increasing the penetration rate of error-free vehicles, reduces the number of accidents and, consequently, increases traffic flow. Fig. 8 studies this behavior in more detail for selected values of T and $d_{\mathbf{s}}$. While these qualitative features are expected, key to our model are exact quantitative characterizations of tradeoffs for each given set of model parameters.

VI. Conclusion

This paper studies safety and efficiency of future traffic systems. We develop a rigorous microscopic model for traffic at intersections. Random misperception of human drivers may trigger accidents. The system is captured by random ordinary differential equations (RODEs) that require specific numerical schemes for their efficient simulation. The proposed setup is general and can be extended to more complex traffic scenarios.

Accidents are a consequence of perceptional errors of human drivers whose probability and size may depend on advanced driver-assistance systems. Our case study clearly illustrates the tradeoff between risk and efficiency. If too many accidents occur, traffic breaks down; but if the safety margins are very large, the system becomes inefficient. Besides delivering these expected qualitative results, the model provides a quantitative simulation lab of future traffic systems. Our model captures both homogeneous vehicles and the coexistence of autonomous vehicles and human drivers. Our techniques can easily be modified to allow for more than two types of drivers.

The proposed approach can also be implemented in more comprehensive models, yet computational costs increase. Other road types, such as roundabouts and multi-lane roads, could be modeled accordingly. In the context of lane-changing, misperception will also imply the occurrence of accidents. The model is capable of characterizing improved or even optimal designs of autonomous vehicles if human drivers that make errors are present and coexist with autonomous vehicles. Another interesting issue are errors of autonomous vehicles, e.g., misclassification of objects; but this important subject requires a more comprehensive model than ours which includes also other participants besides cars.

Finally, future research should, on the one hand, derive surrogate models from the microscopic traffic system. On the other hand, a more detailed analysis of accidents that includes incurred losses may provide additional guidance for the design of traffic systems with autonomous vehicles and for suitable risk management solutions.

Appendix A
Theoretical Existence Result

General existence results for ordinary differential equations can be found in the literature, e.g., the Theorem of Constantin Carathéodory [39], see [34, Chapter 2.1].

Let $B_r(x_0) \subseteq \mathbb{R}^d$ be the open ball with radius $r > 0$ centered in $x_0 \in \mathbb{R}^d$.

Theorem 1 (Carathéodory's Existence Theorem)**.** *Let $f\colon [0,T] \times B_r(x_0) \to \mathbb{R}^d$ such that*

1) *$f(t,x)$ is continuous in x for almost every $t \in [t_0, T]$,*
2) *$f(t,x)$ is Lebesgue measurable in t for all $x \in B_r(x_0)$,*
3) *$|f(t,x)| \leq M(t)$ for all $x \in \mathbb{R}$ and almost every $t \in [t_0, T]$ for some absolutely continuous function $M(t)$.*

Then there exists an absolutely continuous function $x^*\colon [t_0, t_0+\delta] \to \mathbb{R}^d$ with $x^*(t_0) = x_0$ which solves the initial value problem

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(t,x), \qquad x(t_0) = x_0, \qquad x \in \mathbb{R}^d$$

for Lebesgue almost all $t \in [t_0, t_0+\delta]$.

The three conditions are also referred to as *Carathéodory conditions*.

Appendix B
Choice of Parameters

The parameters for our simulations are displayed in Table I.

TABLE I
Parameter Choice for the Scenario

$a_{\max}$	v_{d}	δ	$a_{\min}$	s_0	T	b	l
2.0	10.0	4	-3.5	2.0	$\cdot$	1.67	6
γ	α	β	σ	d_s	t^*	Δt	λ
1/300	1	1	0.2	$\cdot$	10	0.1	1/3

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