Ambiguity sensitive preferences in Ellsberg Frameworks *

Claudia Ravanelli† Gregor Svindland‡

September, 2017

Abstract

We study the market implications of ambiguity sensitive preferences using the α-maxmin expected utility (α-MEU) model. In the standard Ellsberg framework we prove that α-MEU preferences are equivalent to either maxmin, maxmax or subjective expected utility (SEU). We show how ambiguity aversion impacts equilibrium asset prices, and revisit the laboratory experimental findings in Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010). Only when there are three or more ambiguous states, α-MEU, maxmin, maxmax and SEU models induce different portfolio choices. We suggest criteria to discriminate among these models in laboratory experiments and show that ambiguity seeking agents may prevent the existence of market equilibrium. Our results indicate that ambiguity matters for portfolio choice and does not wash out in equilibrium.

JEL Classifications: G11, G12, C92, D53.

Keywords: Ellsberg framework, α-maxmin expected utility model, ambiguity aversion, portfolio choice, market equilibrium.

*We are particularly indebited to Larry Epstein and Bill Zame for insightful discussions. For helpful comments we thank Peter Bossaerts, Pierre Collin-Dufresne, Darrell Duffie, Damir Filipovic, Lorenzo Garlappi, Paolo Ghirardato, Julien Hugonnier, Pablo Koch-Medina, Leonid Kogan, Loriano Mancini and Jean-Charles Rochet, as well as the participants of the 2016 Risk and Stochastics Conference at London School of Economics.

†*Corresponding author:* Claudia Ravanelli, Center for Finance and Insurance, Department of Banking and Finance, University of Zurich, Plattenstr. 22, CH-8032 Zurich, Switzerland. E-mail: claudia.ravanelli@bf.uzh.ch.

‡Gregor Svindland, University of Munich, Mathematics Institute, Theresienstr. 39, D-80333 Munich, Germany. E-mail: gregor.svindland@math.lmu.de.

1 Introduction

Over the past decades the impact of Knightian uncertainty (Knight 1921), or ambiguity, on financial decision making has received significant attention in the academic community.[1] Models with ambiguity averse agents can capture a variety of empirical phenomena such as non-participation, portfolio inertia and excess volatility of asset returns.[2] These models are also supported by experimental laboratory evidence that agents' preferences are heterogeneous and well approximated by ambiguity sensitive preferences with different degrees of ambiguity aversion; e.g., Bossaerts et al. (2010), and Ahn, Choi, Gale, and Kariv (2014).

The workhorse model to study the impact on financial markets of ambiguity aversion has been the maxmin expected utility model (Gilboa and Schmeidler 1989). The α-maxmin expected utility (α-MEU) model generalizes the maxmin model and has a number of appealing features.[3] Being a convex combination of the maxmax (0-MEU) and the maxmin (1-MEU) models, it can represent a large spectrum of preferences, ranging from the ambiguity seeking attitude of the 0-MEU to the ambiguity aversion attitude of the 1-MEU. Assuming that the set of priors that describes the uncertainty of the setting is known, the one-dimensional parameter α can be used to assess the agent's ambiguity aversion. As a consequence, the α-MEU has been used in many theoretical and experimental studies on agents' ambiguity attitudes.[4]

Despite the popularity of the α-MEU model, an in-depth analysis of its portfolio choice and equilibrium asset price implications has not been carried out,[5] although such analysis is central to

[1]See Gilboa and Marinacci (2013), and Machina and Siniscalchi (2014) for recent discussions of ambiguity and ambiguity aversion.

[2]Several studies investigate how agents' nonparticipation may arise in the presence of ambiguity averse agents; see Dow and Werlang (1992), Epstein and Miao (2003), Cao, Wang, and Zhang (2005), Easley and O'Hara (2009), Illeditsch (2011) and Dimmock, Kouwenberg, and Wakker (2016). Studies relating ambiguity aversion to other market phenomena include Chen and Epstein (2002), Uppal and Wang (2003), Trojani and Vanini (2004), Epstein and Schneider (2008), Cao, Han, Hirshleifer, and Zhang (2011), and Boyle, Garlappi, Uppal, and Wang (2012). For a survey on this topic see Epstein and Schneider (2010). Ambiguity averse preferences have been used to address long-standing puzzles in Economics regarding the conflict between efficiency and incentive compatibility, see for e.g. He and Yannelis (2015) and De Castro and Yannelis (2016), and the existence of Rational Expectations Equilibrium, see for e.g. De Castro, Pesce, and Yannelis (2017).

[3]The α-maxmin expected utility is a generalization of the Hurwicz's model introduced by Hurwicz (1951a,b); see also Arrow and Hurwicz (1972) and Jaffray (1988).

[4]Theoretical properties of the α-MEU model have been studied by Ghirardato, Klibanoff, and Marinacci (1998) and Marinacci (2002). For characterizations of subclasses of the α-MEU preferences see Ghirardato, Maccheroni, and Marinacci (2004), Olszewski (2007), Eichberger, Grant, Kelsey, and Koshevoy (2011), and Klibanoff, Mukerji, and Seo (2014). Chen, Katuščák, and Ozdenoren (2007) focus on sealed bid auctions and use the α-MEU to derive the equilibrium bidding strategy for α-MEU bidders. For recent experimental studies see, Ahn et al. (2014), Cubitt, van de Kuilen, and Mukerji (2014) and reference therein.

[5]Bossaerts et al. (2010), and Ahn et al. (2014) derive the α-MEU model portfolio choice in the standard Ellsberg framework. However, as we show in Proposition 3.1, in that framework α-MEU preferences coincide with either maxmin, maxmax or SEU preferences.

understand the attitudes towards ambiguity that a model represents. One reason could be that the α-MEU portfolio optimization is involved. In fact, while the maxmin utility is always concave if the agent is risk averse, this is not in general the case for the α-MEU utility.

This paper theoretically studies the implications for optimal portfolio choice and equilibrium asset prices of the α-MEU model. We carry out this study in a complete Arrow–Debreu market model where the future states of the economy correspond to draws from Ellsberg-type urns. A complete market is an ideal framework to study the optimal portfolio choice implied by a model because agents can attain the desired amount of portfolio risk and ambiguity exposures, given their budget constraints. Moreover, the separation between risky and ambiguous states and interchangeability of the latter in the Ellsberg frameworks allow to pin down the different attitudes towards ambiguity of α-MEU agents as a function of α.

First we consider the standard Ellsberg (1961) framework where the state space consists of three future states of the economy, one risky and two ambiguous. We find that in this setting, α-MEU preferences are equivalent to either maxmin (when $\alpha > 1/2$), maxmax (when $\alpha < 1/2$), or subjective expected utility (SEU) (when $\alpha = 1/2$) preferences.[6] Hereafter we refer to this result as the equivalence result. This shows that the popular standard Ellsberg framework is not the right setting to study α-MEU preferences as a generalization of the maxmin, maxmax and SEU preferences, and has implications for experimental studies. For instance, it rationalizes empirical evidence from recent laboratory experiments carried out in the standard Ellsberg framework that use the α-MEU model to conclude that their experimental evidence point to a substantial heterogeneity in aversion to ambiguity; e.g., Bossaerts et al. (2010), and Ahn et al. (2014).[7] The equivalence result shows that these experimental studies could had come to the same conclusion by using the maxmin model, and varying the size of the set of priors to measure varying degrees of aversion to ambiguity. Moreover, the discovered equivalence between α-MEU with $\alpha > 1/2$ and maxmin preferences allows us to theoretically justify and enhance Bossaerts et al. (2010)'s experimental findings that ambiguity impacts equilibrium asset prices. First, we derive the equilibrium state prices for a market populated by SEU agents and maxmin agents, who optimally choose portfolios with no exposure to ambiguity, and we show theoretically through which channels ambiguity aversion impacts equilibrium state prices. Then we observe that the theoretical rankings of the state-price/state-probability ratios

[6]This result holds true for any number of risky states as long as that there are only two ambiguous states.

[7]Bossaerts et al. (2010) and Ahn et al. (2014) run portfolio choice lab-experiments in the standard Ellsberg framework and provide evidence of considerable heterogeneity in agents' preferences. They find that one half of the agents are well approximated by SEU preferences, while the remaining half has a significant degree of ambiguity aversion and prefers portfolios with no exposure to ambiguity.

fully explain the Bossaerts et al. (2010) empirical rankings.[8] This remarkable matching between theory and data clearly indicates that ambiguity aversion does not wash out in equilibrium.[9]

Next, we consider an extended Ellsberg framework where the state space contains three or more ambiguous states. We show that in this setting the α-MEU preferences do not reduce to and imply portfolio choices that are not observationally equivalent to maxmin, maxmax or SEU preferences. To show this we study the portfolio choice implied by the α-MEU model as a function of $\alpha \in (0, 1)$ when the α-MEU set of priors is fixed to be the one that describes the uncertainty (risk and ambiguity) in the extended Ellsberg framework. We denote this set by $\mathcal{C}_{\max}$ and the corresponding class of models by α-$\mathcal{C}_{\max}$-MEU. Fixing $\mathcal{C}_{\max}$ as the α-MEU set of priors allows to interpret the parameter α as a measure of the agent's aversion towards ambiguity. This allows us to study the α-MEU portfolio choice as a function of the different degrees of ambiguity aversion and make meaningful comparisons between α-$\mathcal{C}_{\max}$-MEU and maxmin preferences using a utility specification common to the two classes of models.[10]

We show that the α-$\mathcal{C}_{\max}$-MEU agents with $\alpha \in (0, 1)$ optimally choose only two types of portfolios: either an unambiguous portfolio (with no exposure to ambiguity, allocating equal wealth to all ambiguous states), or an ambiguous portfolio with a specific exposure to ambiguity: this ambiguous portfolio corresponds to an unambiguous portfolio plus a bet on one of the cheapest ambiguous states. If there is only one ambiguous state with cheapest price, then the optimal portfolio is unique. If there are $n \geq 1$ ambiguous states with cheapest price, then there are n optimal portfolios as the α-$\mathcal{C}_{\max}$-MEU agent finds equally optimal to bet on any of the n cheapest ambiguous states.

The choice between unambiguous and ambiguous optimal portfolios only depends on α and the ratio of the cheapest price to the total sum of prices of the ambiguous states. The larger is α relative to the ratio above, the less the optimal portfolio is exposed to ambiguity. The set of

[8]To rationalize their experimental findings Bossaerts et al. (2010) use a theoretical market model populated by α-MEU agents with $\alpha > 1/2$ and SEU agents. Unaware that in the standard Ellsberg framework α-MEU utilities with $\alpha > 1/2$ reduce to concave maxmin utilities, Bossaerts et al. (2010) do not derive the equilibrium state prices but make conjectures. Our theoretical findings in Section 3.1 complete the Bossaerts et al. (2010) model and show that their experimental findings are much closer to the theory than they could conclude based on their analysis.

[9]To further strengthen this result we remark that while the maxmin portfolio choice explains the fraction of portfolios with no exposure to ambiguity observed in the Bossaerts et al. (2010) experiments, the portfolio choice of a SEU agent cannot, even if the SEU agent is endowed with a non-smooth utility. The intuitive reason is that a kink of a non-smooth SEU utility does not discriminate between risky and ambiguous states, while a kink of the maxmin-utility, consequence of the multiple priors evaluation of the maxmin agent, it does.

[10]Studies of agent's ambiguity aversion based on the α-MEU model typically assume, as we do here, that the set of priors describing the uncertainty of the setting is known; see, e.g., Chen et al. (2007), and Ahn et al. (2014). This assumption has been debated in the literature, but it appears to be necessary to achieve specific behavioral predictions which are amenable to testing in an experimental setting.

state prices for which an α-$\mathcal{C}_{\max}$-MEU agent chooses an unambiguous portfolio increases with the ambiguity aversion parameter α. The limiting case is the 1-$\mathcal{C}_{\max}$-MEU agent who always chooses an unambiguous portfolio.

We show that any α-$\mathcal{C}_{\max}$-MEU agent with an α smaller than $(l-1)/l$, where l is the number of ambiguous states, shows an ambiguity seeking behavior. These agents *always* prefer a portfolio exposed to ambiguity, even when the prices of ambiguous states are all equal and the ambiguous states are thus indistinguishable one another. Only when α is larger than $(l-1)/l$, α-$\mathcal{C}_{\max}$-MEU agents do not exhibit ambiguity seeking behavior, and when the ambiguous state prices are all equal prefer the unambiguous portfolios.[11] In this case, we show that an unambiguous portfolio is also the optimal choice of maxmin agents, irrespectively of the size of their sets of priors.

Our theoretical findings can inform laboratory experiments to disentangle between ambiguity seeking α-$\mathcal{C}_{\max}$-MEU agents and agents who are not ambiguity seeking, and among the latter, between α-$\mathcal{C}_{\max}$-MEU and maxmin agents. We propose a multiple-stage experiment. In the first stage, setting the prices of ambiguous states all equal allows to identify the ambiguity seeking agents from their portfolio choices. Subsequent stages only involve the non-ambiguity seeking agents, and exploit the fact that optimal portfolios of α-$\mathcal{C}_{\max}$-MEU agents are not unique, while the optimal portfolio of maxmin agents is typically unique.

Finally, we study the equilibrium in a market populated by ambiguity sensitive and SEU agents. We show that the existence of equilibrium depends on whether ambiguity seeking agents are or are not in the market. More precisely, we find that the ambiguity seeking α-$\mathcal{C}_{\max}$-MEU agents may prevent the existence of market equilibrium that otherwise exists if together with SEU agents the non-ambiguity seeking α-$\mathcal{C}_{\max}$-MEU or maxmin agents populate the market. Intuitively, ambiguity seeking agents may take positions that cannot be offset by the SEU agents.

The structure of the paper is as follows. Section 2 introduces the setup. Section 3 shows that in the standard Ellsberg framework α-MEU preferences coincide with maxmin, maxmax and SEU preferences, derives equilibrium state prices, and revisits the experimental findings in Bossaerts et al. (2010). Section 4 studies the optimal portfolio choice of the α-$\mathcal{C}_{\max}$-MEU model in the extended Ellsberg framework, and the different attitudes towards ambiguity. Section 5 studies the impact of ambiguity seeking behaviors on equilibrium asset prices. Section 6 concludes. The

[11] In contrast to the standard Ellsberg framework, in the extended Ellsberg frameworks (i.e. when $l \geq 3$) the value of α that separates ambiguity seeeking from ambiguity averse agents is not anymore $\alpha = 1/2$ but $\alpha = (l-1)/l$: when $l = 3$, $\alpha = (l-1)/l = 2/3$ and increases towards 1 when l increases. Moreover, $(l-1)/l$-$\mathcal{C}_{\max}$-MEU preferences do not reduce to SEU preferences; see Section 4.3.

Appendix collects proofs and technical results.

2 Setup

The utility of an α-MEU agent from some state dependent wealth $w = (w_\sigma)_{\sigma \in S}$ is

$$U(w) = \alpha \min_{\pi \in \mathcal{C}} \sum_{\sigma \in S} u(w_\sigma)\, \pi_\sigma + (1-\alpha) \max_{\pi \in \mathcal{C}} \sum_{\sigma \in S} u(w_\sigma)\, \pi_\sigma \tag{2.1}$$

where $u : \mathbb{R} \to \mathbb{R}$ is a utility function, the set of priors $\mathcal{C}$ is a closed and convex set on the finite state space S, and α can take any value between $[0, 1]$. For $\alpha = 1$, (2.1) reduces to the maxmin-expected utility (1-MEU) model, for $\alpha = 0$ to the maxmax-expected utility (0-MEU) model. All utility functions $u : \mathbb{R} \to \mathbb{R}$ are differentiable, strictly concave and strictly increasing. To keep the analysis tractable we assume that u is defined on the whole real line. The majority of results in this paper (e.g., the α-MEU portfolio characterization in Proposition 4.1) holds true also when u has a bounded domain, as long as the set of feasible portfolios remain convex and the utility differentiable.[12]

The market model considered in this paper is an Arrow–Debreu complete market for contingent claims with two dates, $t = 0$ and $t = 1$. S is the finite state space containing all possible states of the economy at time $t = 1$, and $\mid S \mid$ is the number of states. At time $t = 0$ the agents face both uncertainty (risk) and ambiguity since they neither know which state in S will realize at time $t = 1$ (uncertainty), nor what is the probability of the occurrence of some of the states in S (ambiguity). For any state there is an Arrow security traded in the market which pays at time $t = 1$ one unit of currency in that state and nothing in the other states. Pricing rules $p = (p_\sigma)_{\sigma \in S} \in \mathbb{R}_+^{|S|}$ are normalized so that the price of the risk-free and unambiguous portfolio $w = (1, \ldots, 1)$ is 1, that is $\sum_{s=1}^{|S|} p_i = 1$.

Given N agents in the market, each agent n is characterized by an initial endowment $e^n \in \mathbb{R}^{|S|}$, where the ith coordinate of e^n corresponds to the number of Arrow securities that pay in the state i, and by a criterion U^n representing her preferences, $n = 1, \ldots, N$. The total endowment in the market is $W := \sum_1^N e^n = (W_1, \ldots, W_{|S|})$, where $:=$ denotes definition. Let $\cdot$ denote the scalar product $x \cdot y = \sum_{i=1}^{|S|} x_i y_i$, $x, y \in \mathbb{R}^{|S|}$. Given the pricing rule p on the Arrow securities, a portfolio $w^n = (w_\sigma^n)_{\sigma \in S} \in \mathbb{R}^{|S|}$ is said to be optimal for agent n if w^n satisfies the budget constraint

[12] These properties can be insured, for instance, by requiring that the feasible portfolios are in the interior of the utility domain.

$p \cdot w^n \leq p \cdot e^n$ and maximizes the utility U^n over all portfolios $w \in \mathbb{R}^{|S|}$ subject to the budget constraint $p \cdot w \leq p \cdot e^n$, i.e.

$$U^n(w^n) = \max\{U^n(w) \mid w \in \mathbb{R}^{|S|}, p \cdot w \leq p \cdot e^n\}.$$

An equilibrium $(p; w^1, \ldots, w^N)$ consists of a pricing rule p and individual portfolio choices w^n such that

- for each $n = 1, \ldots, N$ the portfolio w^n is optimal for agent n given the pricing rule p, and
- the market clears: $\sum_1^N w^n = \sum_1^N e^n$.

3 Standard Ellsberg framework

Throughout this section we consider a standard Ellsberg framework, that is a state space $S = \{R, G, B\}$ where the states correspond to draws from the Ellsberg (1961) urn. The probability of the state R (red) is known and equal to $\pi_R \in (0, 1)$, while the probabilities of the two ambiguous states G (green) and B (blue) are unknown. Any closed convex set of priors $\mathcal{D}$, consistent with the above information on the Ellsberg framework, can be written as

$$\mathcal{D} = \{(\pi_R, q, 1 - q - \pi_R) : q \in [a, b]\} \tag{3.1}$$

where π_R, q, and $1 - q - \pi_R$ are the probability weights on the states R, G, and B, respectively, corresponding to a given prior in $\mathcal{D}$, and $0 \leq a \leq b \leq 1 - \pi_R$. Thus, any α-MEU utility U in (2.1) on the portfolio $w = (w_R, w_G, w_B) \in \mathbb{R}^3$ reads as

$$\begin{aligned} U(w) \quad = \quad & \alpha \min_{q \in [a,b]} [\pi_R\, u(w_R) + q\, u(w_G) + (1 - q - \pi_R)\, u(w_B)] + \\ & (1 - \alpha) \max_{q \in [a,b]} [\pi_R\, u(w_R) + q\, u(w_G) + (1 - q - \pi_R)\, u(w_B)] \end{aligned} \tag{3.2}$$

for some $\alpha \in [0, 1]$.

3.1 Equivalence result

In the following Proposition 3.1, we show that α-MEU preferences are equivalent to either 1-MEU, 0-MEU or SEU preferences. The proof is provided in Appendix B.

Proposition 3.1. *Consider the utility U in (3.2) and let $c := \alpha\, a + (1-\alpha) b$ and $d := (1-\alpha) a + \alpha\, b$.*

(i) *If $\alpha > 1/2$, then U is a maxmin expected utility (1-MEU), i.e.*

$$U(w) = \min_{q \in [c,d]} \left[\pi_R\, u(w_R) + q\, u(w_G) + (1 - q - \pi_R)\, u(w_B)\right] \tag{3.3}$$

with set of priors $\mathcal{C} = \{(\pi_R, q, 1 - q - \pi_R) : q \in [c, d]\} \subset \mathcal{D}$.

(ii) *If $\alpha = 1/2$, then U is a subjective expected utility (SEU) with subjective prior*

$$(\pi_R, (a + b)/2, 1 - \pi_R - (a + b)/2). \tag{3.4}$$

(iii) *If $\alpha < 1/2$, then U is a maxmax expected utility (0-MEU), i.e.*

$$U(w) = \max_{q \in [d,c]} \left[\pi_R\, u(w_R) + q\, u(w_G) + (1 - q - \pi_R)\, u(w_B)\right] \tag{3.5}$$

with set of priors $\mathcal{C} = \{(\pi_R, q, 1 - q - \pi_R) : q \in [d, c]\} \subset \mathcal{D}$.

Proposition 3.1 shows that any α-MEU utility with $\alpha > 1/2$ ($\alpha < 1/2$) and a generic set of priors $\mathcal{D}$ is equivalent to a unique maxmin utility (respectively, maxmax utility) over a set of priors $\mathcal{C}$, which is smaller than $\mathcal{D}$, and univocally characterized by α and $\mathcal{D}$.[13] Consequently,

- the α-MEU preferences in the standard Ellsberg framework are indistinguishable from maxmin, maxmax or SEU preferences
- doing comparative statics with respect to $\alpha \in (1/2, 1)$ ($\alpha \in (0, 1/2)$) in the α-MEU model with a fixed set of prior is equivalent to doing comparative statics with respect to the size of the set of priors in the maxmin (respectively, maxmax) model.

These findings provide new insights into the implications of ambiguity for portfolio choice and have implications for experimental studies. For instance, they show that the standard Ellsberg framework is not the right setting to study the α-MEU model if one wants to use this model as a generalization of the maxmin, maxmax or SEU preferences.[14] Moreover, they clarify some recent

[13] The set of priors $\mathcal{C}$ equals $\mathcal{D}$ when $\alpha = 1$ ($\alpha = 0$), and when α decreases (increases) to $1/2$ shrinks up to only containing the prior (3.4).

[14] Section 4 shows that when there are more than two ambiguous states the equivalence result does not hold anymore: the α-MEU model induces different portfolio choice and expresses different attitude towards ambiguity than the maxmin maxmin, maxmax and SEU models.

experimental studies carried out in the standard Ellsberg framework in which the α-MEU model is used to conclude that the experiment outcomes suggest a substantial heterogeneity in aversion to ambiguity; e.g. Bossaerts et al. (2010), and Ahn et al. (2014). Proposition 3.1 shows that these studies could have come to the same conclusions by using maxmin preferences and varying the size of the set of priors instead of α to measure varying degrees of aversion to ambiguity. For the implications of the equivalence result on equilibrium asset prices see Section 3.4.

The converse of Proposition 3.1 was already known from Siniscalchi (2006), namely that a maxmin model with a given set of priors $\mathcal{C}$ can be rewritten as less parsimonious α-MEU models, with set of priors $\mathcal{D}$ larger than $\mathcal{C}$, for many different $(\alpha, \mathcal{D})$.

The α-MEU preferences coincide with maxmin, maxmax, or SEU preferences also in a state space setting with more than one risky states, or with no risky states, as long as there are only two ambiguous states. When there are no risky states, Proposition 3.1 holds true by setting $\pi_R = 0$. When there are $m \geq 1$ risky states, $R_1, \ldots, R_m$, with known probabilities $\pi_{R_i} \in (0,1)$ which satisfy $\sum_{i=1}^{m} \pi_{R_i} < 1$, Proposition 3.1 holds true by replacing the prior in (3.4) by the prior $(\pi_{R_1}, \ldots, \pi_{R_m}, (a+b)/2, 1 - \sum_{i=1}^{m} \pi_{R_i} - (a+b)/2)$, and $\pi_R u(w_R)$ in (3.3) and (3.5) by $\sum_{i=1}^{m} \pi_{R_i} u(w_{R_i})$.

3.2 Market equilibrium with ambiguity averse and SEU agents

Motivated by recent experimental evidences that investor's preferences are well approximated by ambiguity averse and SEU preferences, we study a simple market model populated by maxmin agents (or equivalently by α-MEU agent with $\alpha > 1/2$, see Proposition 3.1 (i)) and SEU agents. We derive equilibrium asset prices and show theoretically how ambiguity aversion impacts equilibrium asset prices.

Let denote by $w = (w_R, w_G, w_B)$ the optimal portfolio of a maxmin agent and by $y = (y_R, y_G, y_B)$ the optimal portfolio of a SEU agent. Depending on the distribution of the total endowment $W = (W_R, W_G, W_B)$ in the market, only particular rankings of state-price/state-probability ratios can occur in equilibrium. The interesting case to study is when maxmin agents take an unambiguous portfolio, i.e. $w_G = w_B$, and the total endowment is $W_G \neq W_B$. Without loss of generality in the following we assume that $W_G > W_B$.

Appendix A provides the proof of Proposition 3.2 and a concise treatment of maxmin and SEU portfolio choice.

Proposition 3.2. *Suppose the market is in equilibrium and populated by maxmin agents (equivalently α-MEU preferences with $\alpha > 1/2$) who take unambiguous portfolios and SEU agents with prior $\pi = (\pi_R, \pi_G, \pi_B)$, with $\pi_R, \pi_G, \pi_B > 0$. Denote by $W = (W_R, W_G, W_B) \in \mathbb{R}^3$ the total endowment of the market.*

1) *If $W_R > W_G > W_B$, then two rankings of the state-price/state-probability ratios are possible:*

$$\frac{p_B}{\pi_B} > \frac{p_R}{\pi_R} > \frac{p_G}{\pi_G} \tag{3.6}$$

and the optimal portfolios y of any SEU agent and w of any maximin agent satisfy

$$y_G > y_R > y_B \quad \text{and} \quad w_R > w_G = w_B.$$

The other possible ranking is:

$$\frac{p_B}{\pi_B} > \frac{p_G}{\pi_G} > \frac{p_R}{\pi_R} \quad \left(or \quad \frac{p_B}{\pi_B} > \frac{p_G}{\pi_G} = \frac{p_R}{\pi_R} \right) \tag{3.7}$$

and the optimal portfolios y of any SEU agent and w of any maxmin agent satisfy

$$y_R > y_G > y_B \quad (or \quad y_R = y_G > y_B) \quad \text{and} \quad w_R > w_G = w_B.$$

2) *If $W_G > W_R > W_B$, then the only possible ranking of the state-price/state-probability ratios is:*

$$\frac{p_B}{\pi_B} > \frac{p_R}{\pi_R} > \frac{p_G}{\pi_G} \tag{3.8}$$

and the optimal portfolios y of any SEU agent and w of any maxmin agent satisfy

$$y_G > y_R > y_B \quad \text{and} \quad w_R > w_G = w_B \quad or \quad w_R < w_G = w_B.$$

3) *If $W_G > W_B > W_R$, then two rankings of the state-price/state-probability ratios are possible:*

$$\frac{p_B}{\pi_B} > \frac{p_R}{\pi_R} > \frac{p_G}{\pi_G} \tag{3.9}$$

and the optimal portfolios y of any SEU agent and w of any maximin agent satisfy

$$y_G > y_R > y_B \quad \textit{and} \quad w_R < w_G = w_B.$$

The other possible ranking is:

$$\frac{p_R}{\pi_R} > \frac{p_B}{\pi_B} > \frac{p_G}{\pi_G} \quad \left(\textit{or} \quad \frac{p_R}{\pi_R} = \frac{p_B}{\pi_B} > \frac{p_G}{\pi_G} \right) \tag{3.10}$$

and the optimal portfolios y of any SEU agent and w of any maxmin agent satisfy

$$y_G > y_B > y_R \quad (\textit{or} \quad y_G > y_B = y_R) \quad \textit{and} \quad w_R < w_G = w_B.$$

Proposition 3.2 shows that when the total endowment $W = (W_R, W_G, W_B)$ satisfies $W_R > W_G > W_B$ or $W_G > W_B > W_R$, two rankings of the state-price/state-probability ratios of the SEU agents are possible in equilibrium. In the examples of Section 3.3 we show that all these rankings can indeed occur.

The actual occurrence of ranking (3.6) when $W_R > W_G > W_B$, and of ranking (3.9) when $W_G > W_B > W_R$, show that ambiguity aversion strongly impacts equilibrium prices. In these cases, the SEU agents in the market and the SEU representative agent (who rationalizes the market equilibrium) rank state-price/state-probability ratios in equilibrium differently.[15] Since the maxmin agents take an unambiguous portfolio, i.e. $w_G = w_B$, and $W_G > W_B$, the market clearing implies that SEU agents have to hold portfolios which in aggregate pay strictly more on state G than on state B[16]. To induce the SEU agents to clear the supply difference $W_G - W_B$, the price of the Arrow security G in larger supply has to be comparatively lower and the price of the Arrow security in lower supply B has to comparatively higher than, for instance, in a market populated only by SEU agents sharing the same prior. When the supply differences $W_G - W_B$ will be above a certain level, the SEU agents will have to hold in equilibrium a state dependent portfolio that does not rank as the state dependent total endowment W, and consequently their state-price/state-probability ratios will not be ranked opposite to W. For instance, when $W_G > W_B > W_R$ and $W_G - W_B$ is "too

[15] Rankings (3.6) and (3.9) of the state-price/state-probability ratio of SEU agents in the market are not opposite to the ranking of the corresponding total endowments. By contrast, the state-price/state-probability of the SEU representative agent who rationalizes the market equilibrium is ranked opposite to total endowment: the representative agent has to hold the total endowment of the economy as optimal portfolio, thus (A.3) has to hold.

[16] This occurs if and only if $\frac{p_B}{\pi_B} > \frac{p_G}{\pi_G}$ (See (A.3) in Appendix A) and excludes all state-price/state-probability rankings in which $\frac{p_B}{\pi_B} \leq \frac{p_G}{\pi_G}$.

large" compared to $W_B - W_R$, the optimal portfolio of the SEU agents will be $y_G > y_R > y_B$ and the ranking (3.9). The examples in Section 3.3 illustrate the mechanism through which ambiguity averse agent impacts prices in the CARA and Quadratic cases.

An ambiguity averse representative agent who rationalizes the market equilibrium may also be possible. However, the representative agent will need to be less ambiguity averse (that is, to have a smaller set of priors) than the maxmin agents acting in the market.[17]

3.3 Illustrating equilibrium results

We now illustrate Proposition 3.2 in the case of exponential and quadratic utilities. We recall that the market is populated by SEU agents with prior $\pi = (\pi_R, \pi_G, \pi_B)$, with $\pi_R, \pi_G, \pi_B > 0$, and by maxmin agents in (3.3) who are sufficiently ambiguity averse to hold an unambiguous portfolio, i.e. $w_G = w_B$.[18] The total endowment $W = (W_R, W_G, W_B)$ is such that $W_G > W_B$.

3.3.1 CARA utility

There are L SEU agents and M maxmin agents, all having exponential utilities $u(z) = 1 - \frac{e^{-\delta z}}{\delta}$. Let $\delta = \beta$ and $\delta = \gamma$ be the risk aversion parameter of the SEU agents and maxmin agents, respectively. Then, the equilibrium state prices are:

$$\begin{aligned}
p_R &= \frac{\pi_R}{\pi_R + e^{\frac{\beta\gamma(W_R - W_G)}{\beta M + \gamma L}} \pi_G^{\frac{\gamma L}{\beta M + \gamma L}} q^{\frac{\beta M}{\beta M + \gamma L}} + e^{\frac{\beta\gamma(W_R - W_B)}{\beta M + \gamma L}} \pi_B^{\frac{\gamma L}{\beta M + \gamma L}} (1 - \pi_R - q)^{\frac{\beta M}{\beta M + \gamma L}}} \\
p_G &= \frac{\pi_G^{\frac{\gamma L}{\beta M + \gamma L}} q^{\frac{\beta M}{\beta M + \gamma L}}}{\pi_G^{\frac{\gamma L}{\beta M + \gamma L}} q^{\frac{\beta M}{\beta M + \gamma L}} + \pi_R e^{\frac{\beta\gamma(W_G - W_R)}{\beta M + \gamma L}} + e^{\frac{\beta\gamma(W_G - W_B)}{\beta M + \gamma L}} \pi_B^{\frac{\gamma L}{\beta M + \gamma L}} (1 - \pi_R - q)^{\frac{\beta M}{\beta M + \gamma L}}} \\
p_B &= \frac{\pi_B^{\frac{\gamma L}{\beta M + \gamma L}} (1 - \pi_R - q)^{\frac{\beta M}{\beta M + \gamma L}}}{\pi_B^{\frac{\gamma L}{\beta M + \gamma L}} (1 - \pi_R - q)^{\frac{\beta M}{\beta M + \gamma L}} + \pi_R e^{\frac{\beta\gamma(W_B - W_R)}{\beta M + \gamma L}} + e^{\frac{\beta\gamma(W_B - W_G)}{\beta M + \gamma L}} \pi_G^{\frac{\gamma L}{\beta M + \gamma L}} q^{\frac{\beta M}{\beta M + \gamma L}}}
\end{aligned}$$

[17]Suppose there exists a maxmin representative agent characterized by the set of priors $\{(\pi_R, q, 1 - \pi_R - q) \mid q \in [c_R, d_R]\}$ who rationalizes the market equilibrium. Let $\{(\pi_R, q, 1 - \pi_R - q) \mid q \in [c, d]\}$ be the set of priors of the maxmin agents in the market. Since the maxmin agents in the market choose the unambiguous portfolio and the representative agent has to hold the total endowment W of the economy as optimal portfolio, (A.5) implies that: If $W_G > W_B$ then,

$$\frac{c}{1 - \pi_R - c} \leq \frac{p_G}{p_B} < \frac{c_R}{1 - \pi_R - c_R},$$

has to hold true and thus $c_R > c$. With a similar reasoning, if $W_G < W_B$, then $d_R < d$ has to hold true.

[18]From (A.5) and the results that follow in this section (specifically equalities (3.12)) one can see that maxmin agents hold an unambiguous portfolio, i.e. $w_G = w_B$ if and only if q in (3.11) satisfies $c \leq q < \pi_G < d$.

where

$$q := \pi_G \frac{\pi_G + \pi_B}{\pi_G + \pi_B e^{\frac{\beta}{L}(W_G - W_B)}}. \tag{3.11}$$

The dependence of q on $W_G - W_B$ illustrates one channel through which ambiguity aversion affects asset prices. The impact of an increases of $\frac{\beta}{L}(W_G - W_B)$ on the securities prices that pay in the ambiguous states is clear: an increase of $\frac{\beta}{L}(W_G - W_B)$ decreases q and increases $(1 - \pi_R - q)$, and consequently decreases p_G, and increases p_B thus making the SEU agents to absorb the imbalance $W_G - W_B$.[19] The equilibrium price ratios

$$\begin{aligned}
\frac{p_G}{p_B} &= \frac{\pi_G}{\pi_B} e^{-\frac{\beta}{L}(W_G - W_B)} \\
\frac{p_G}{p_R} &= \frac{\pi_G}{\pi_R} \left(\frac{\pi_G + \pi_B}{\pi_G + \pi_B e^{\frac{\beta}{L}(W_G - W_B)}} \right)^{\frac{\beta M}{\beta M + \gamma L}} e^{-\frac{\beta\gamma}{\beta M + bL}(W_G - W_R)} \\
\frac{p_B}{p_R} &= \frac{\pi_B}{\pi_R} \left(\frac{\pi_G + \pi_B}{\pi_G e^{\frac{-\beta}{L}(W_G - W_B)} + \pi_B} \right)^{\frac{\beta M}{\beta M + \gamma L}} e^{-\frac{\beta\gamma}{\beta M + \gamma L}(W_B - W_R)}
\end{aligned} \tag{3.12}$$

show that all rankings of the state-price/state-probability ratios that are possible according to Proposition 3.2 can indeed occur. For instance, consider the case $W_G > W_B > W_R$. The first two equations in (3.12) show that always $\frac{p_B}{\pi_B} > \frac{p_G}{\pi_G}$ and $\frac{p_R}{\pi_R} > \frac{p_G}{\pi_G}$. The third equation in (3.12) shows that both $\frac{p_B}{\pi_B} > \frac{p_R}{\pi_R}$ and $\frac{p_B}{\pi_B} < \frac{p_R}{\pi_R}$ can occur and, consequently, the corresponding rankings (3.9) and (3.10).

The above formulae show that the same equilibrium could also be obtained in a market populated by expected utility maximizers only, but this under the condition that together with the L SEU agents with prior (π_R, π_G, π_B) the remaining M agents are expected utility maximizers with the *unusual* prior $(\pi_R, q, 1 - \pi_R - q)$. The prior $(\pi_R, q, 1 - \pi_R - q)$ is unusual because depends on (and thus changes with) the aggregate endowment on the ambiguous states, and on the number and the risk aversion of the different agents acting in the market.[20] This prior has to be such to make the M expected utility maximizers behave as maxmin agents who always choose the unambiguous portfolio. Bossaerts et al. (2010, Section 4) also argue against such priors.

[19] Depending on the particular rank of the total endowment $W = (W_R, W_B, W_G)$, the price of the Arrow security that pays in the state R will increase or decrease with $\frac{\beta}{L}(W_G - W_B)$.

[20] The fact that q does not depend neither on the number of M of maxmin agents nor on their risk aversion γ is a peculiarity of the exponential utility.

3.3.2 Quadratic utility

The quadratic utility with parameter $c > 0$ reads

$$u_c(x) = \begin{cases} x - cx^2/2, & x \le 1/c \\ 1/(2c), & x > 1/c \end{cases} \tag{3.13}$$

and feasible portfolios live on the strictly increasing part of the utility function. Suppose the SEU agents have utility u_a, the maxmin agents utility u_b and $a, b > 0$. The fixed point equations for the equilibrium prices are

$$\begin{aligned} p_R &= \frac{\pi_R(c - W_R)}{c - \pi \cdot W} \\ p_G &= \frac{\pi_G(c - W_G)}{c - \pi \cdot W}\left(\frac{D - (\frac{1}{b} - X^{\mathrm{MEU}})(c - \pi \cdot W)(1 + \frac{W_G - W_B}{c - W_G}\frac{\pi_B}{1-\pi_R})}{D - (\frac{1}{b} - X^{\mathrm{MEU}})(c - \pi \cdot W)}\right) \\ p_B &= \frac{\pi_B(c - W_B)}{c - \pi \cdot W}\left(\frac{D - (\frac{1}{b} - X^{\mathrm{MEU}})(c - \pi \cdot W)(1 - \frac{W_G - W_B}{c - W_B}\frac{\pi_G}{1-\pi_R})}{D - (\frac{1}{b} - X^{\mathrm{MEU}})(c - \pi \cdot W)}\right) \end{aligned}$$

where $c := \frac{1}{a} + \frac{1}{b}$, $D := \pi_R(c - W_R)^2 + (1 - \pi_R)(c - \frac{\pi_G W_G + \pi_B W_B}{1 - \pi_R})^2$ and X^{MEU} is the initial wealth of the maxmin agents. Note that p_G is lower (p_B is higher) than the price $\frac{\pi_G(c - W_G)}{c - \pi \cdot W}$ (respectively $\frac{\pi_B(c - W_B)}{c - \pi \cdot W}$) that would result in a market equilibrium with SEU agents sharing the same prior (π_R, π_G, π_B).

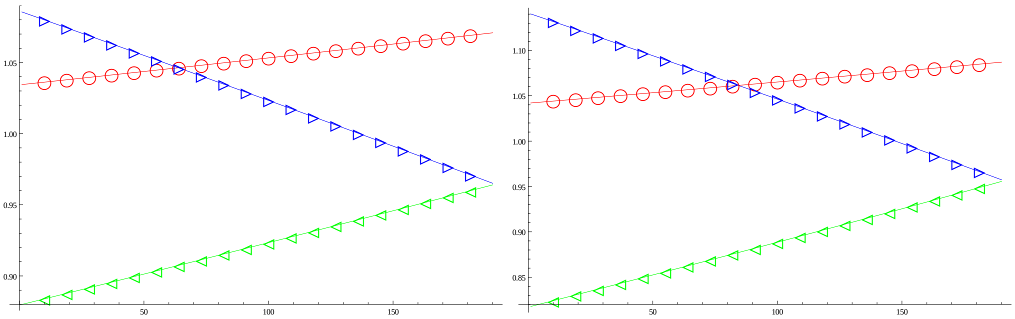
Let $W_G > W_B > W_R$. Figure 1 shows the state-price/state-probability ratios of the equilibrium prices as a function of the difference $W_B - W_R$, computed for fixed $W_G = 272$ and $W_R = 81$[21]. The parameters a and b in (3.13) are set to 0.001 in the left graph, and to $a = 0.0015$ and $b = 0.001$ in the right graph.

In both cases there is a clear change of rankings of state-price/state-probability ratios: as $W_B - W_R$ increases, the ranking switches from (3.9) to (3.10).

3.4 Revisiting laboratory experimental findings

Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010) run a series of laboratory experiments that reproduce a competitive financial market in the standard Ellsberg framework. The comparison between the experimental cross sectional distribution of the security holdings and empirical state-

[21] These values of the aggregate endowment W are the same values used by Bossaerts et al. (2010) in one of their experiments that we discuss in Section 3.4 and which empirical rankings are summarized in Figure 2 in our paper.



(a) $a = b = 0.001$.

(b) $a = 0.0015$ and $b = 0.001$.

Figure 1: State-price/state-probability ratios (y-axis) of the equilibrium prices as a function of the difference $W_B - W_R$ (x-axis), computed for fixed $W_G = 272$ and $W_R = 81$, as in Figure 2. The line marked with circles represents p_R/π_R, the one marked with arrows pointing to the right represents p_B/π_B, and the one marked with arrows pointing to the left represents p_G/π_G. The SEU prior is $\pi_R = \pi_G = \pi_B = 1/3$. The parameters a and b in (3.13) are set to 0.001 in the left graph, and to $a = 0.0015$ and $b = 0.001$ in the right graph.

price/state-probability ratios with and without ambiguity, provide clear evidence that ambiguity aversion matters for portfolio choices and equilibrium prices, and does not wash out in aggregate. To support their experimental findings Bossaerts et al. (2010) use a theoretical market model involving ambiguity averse α-MEU agents with $\alpha > 1/2$ and SEU agents. Bossaerts et al. (2010) do not derive the equilibrium prices but make conjectures about the equilibrium state-price/state-probability ratio. Our theoretical findings in Section 3.1 complete the Bossaerts et al. (2010) model [22] and show that their experimental findings are much closer to the theory than they could conclude just from the analysis in their paper.[23]

In the following we show that the rankings of the state-price/state-probability ratios in Proposition 3.2 resulting in a market equilibrium with maxmin agents who optimally choose unambiguous portfolios and SEU agents, fully explain and theoretically justify all empirical rankings documented by Bossaerts et al. (2010).

Bossaerts et al. (2010) summarize their experimental findings about equilibrium asset prices in Figures 6–8; Bossaerts et al. (2010, pages 1349 and 1350). These figures show the empirical distribution functions of the state-price/state-probability ratios obtained from experimental sessions with different total endowments.

[22]Note that the market model for which we derive the theoretical rankings is the same theoretical market model proposed by Bossaerts et al. (2010) to explain the experimental market. Relative to Bossaerts et al. (2010), from the equivalence result (Proposition 3.1) in addition we know that in the standard Ellsberg framework risk averse α-MEU preferences with $\alpha > 1/2$ are equivalent maxmin preferences with concave utility.

[23]The analysis in the Bossaerts et al. (2010) not always could explain the experimental findings, as discussed by the authors; see the Conclusion in Bossaerts et al. (2010).

Figure 6 in Bossaerts et al. (2010), where the total endowment $W = (W_R, W_G, W_B)$ is such $W_G > W_R > W_B$, provides evidence of one ranking of the empirical state-price/state-probability ratio, which is exactly the ranking (3.8) predicted by our Proposition 3.2. Proposition 3.2 confirms the conjecture in Bossaerts et al. (2010, page 1339) that ranking (3.8) is more likely to occur when $W_G > W_R > W_B$, which is indeed the only theoretically possible ranking. Moreover, Proposition 3.2 shows that the theoretically possible rankings of the state-price/state-probability ratio do depend on the ranking of W_R with respect to W_G and W_B.

Figure 2 below is a copy of Figure 8, right panel, in Bossaerts et al. (2010) where $W_G > W_B > W_R$. The experimental findings summarized in Figure 2 provide evidence of two rankings:

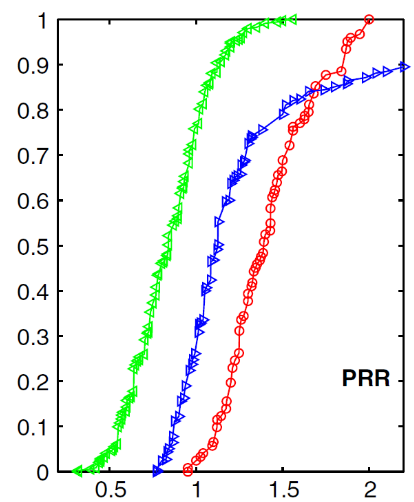


Figure 2: Empirical distribution functions of state-price/state-probability ratios from the experimental session of eight trading periods in Bossaerts et al. (2010) with $W_G = 272$, $W_B = 162$, and $W_R = 81$. The distribution function with circles is for p_R/π_R; the one with arrows pointing to the right is for p_B/π_B; the one with arrows pointing to the left is for p_G/π_G. This figure is a copy of Figure 8, right panel, in Bossaerts et al. (2010).

$p_R/\pi_R > p_B/\pi_B > p_G/\pi_G$, and $p_B/\pi_B > p_R/\pi_R > p_G/\pi_G$.[24] Remarkably, Proposition 3.2 shows that these are exactly the two rankings possible, namely (3.9) and (3.10), when $W_G > W_B > W_R$.

The proof of Proposition 3.2 further shows that the ranking (3.10) prevails when $W_B - W_R$ is large enough to imply an optimal portfolio of the SEU agents with more Arrow securities that pay in the ambiguous state B than in the risky state R. This provides a potential explanation why in Figure 2 the prices do not settle in favor of one of the two rankings: the values W_R, W_G, and

[24] Bossaerts et al. (2010, Page 1351) report that "the rankings appear anomalous," because they only expect to see the second ranking (Bossaerts et al. 2010, Page 1339).

W_B in the experimental section in Figure 2 are close to the point at which the change from (3.9) to (3.10) takes place. Example 3.3.2 illustrates this point: When $W_B - W_R$ is approximately 81, the switch of the rankings occurs as in Figure 1. This confirms that to observe only one ranking of state-price/state-probability ratios in laboratory experiment, the difference in aggregate wealth $W_B - W_R$ should be chosen either relatively large or small.

Bossaerts et al. (2010) perform other experimental sessions in which $W_G > W_B > W_R$, summarized in Figure 7 in Bossaerts et al. (2010). Although the most common ranking of state-price/state-probability ratios is (3.9), the empirical distribution functions of p_R/π_R and p_B/π_B are very close. Proposition 3.2 predicts that to observe a clear separation of the rankings in (3.9) and (3.10), the aggregate wealth W_B should be chosen closer to W_R or W_G, respectively.

4 Extended Ellsberg framework

In this section we consider an extended Ellsberg framework, that is a state space S where the future states of the economy correspond to draws from an extended Ellsberg (1961) urn with m risky states with known probability, and $l \geq 3$ ambiguous states. $A \subset S$ denotes the set that contains the ambiguous states, thus $|A| = l$. The known probabilities $\pi_R \in (0,1)$ of risky states $R \in S \setminus A$ satisfy $\sum_{R \in S \setminus A} \pi_R < 1$.

In this setting the α-MEU model (2.1) represents a large spectrum of preferences that include, but do not reduce to, maxmin, maxmax, SEU preferences. In fact, when $l \geq 3$, a α-MEU utility with $\alpha \in (0,1)$ cannot be in general rewritten as a maxmin, maxmax or SEU utility, although there are specific sets of priors $\mathcal{C}$ for which this is still the case. The following proposition provides examples of α-MEU utilities with $\alpha \in (0,1)$ that reduce to maxmin, maxmax or SEU utility. The proof is provided in Appendix B.

Proposition 4.1. *Consider a set of priors of the form*

$$\mathcal{C} = \left\{ q \in \mathbb{R}^{k+l} \mid q_\sigma = \pi_\sigma,\ \sigma \in S \setminus A \text{ and } q_\sigma \in [a_\sigma, b_\sigma],\ \sigma \in A \setminus \{\eta\}, \text{ and } q_\eta = 1 - \sum_{\sigma \neq \eta} q_\sigma \right\} \tag{4.1}$$

where $\eta \in A$ is an arbitrary but fixed ambiguous state, $0 \leq a_\sigma < b_\sigma$, $\sigma \in A \setminus \{\eta\}$ and $\sum_{\sigma \in A \setminus \{\eta\}} b_\sigma \leq 1 - \sum_{\sigma \in S \setminus A} \pi_\sigma$. Then, the α-MEU utility U with set of priors $\mathcal{C}$ equals

$$U(w) = u(w_\eta) + \sum_{\sigma \in S \setminus A} (u(w_\sigma) - u(w_\eta))\pi_\sigma + \sum_{\sigma \in A \setminus \{\eta\}} (u(w_\sigma) - u(w_\eta))^+ c_\sigma - (u(w_\sigma) - u(w_\eta))^- d_\sigma$$

where $c_\sigma := \alpha a_\sigma + (1-\alpha)b_\sigma$, *and* $d_\sigma := \alpha b_\sigma + (1-\alpha)a_\sigma$, $\sigma \in A \setminus \{\eta\}$.

(i) *If* $\alpha > 1/2$, *then* $c_\sigma < d_\sigma$ *for all* $\sigma \in A \setminus \{\eta\}$ *and* U *is a maxmin expected utility* (1*-MEU) with set of priors* $\widehat{\mathcal{C}} = \{q \in \mathcal{C} : q_\sigma \in [c_\sigma, d_\sigma]\}$.

(ii) *If* $\alpha = 1/2$, *then* U *is a subjective expected utility (SEU) with subjective prior* $\hat{q}$ *satisfying* $\hat{q}_\sigma = \pi_\sigma$ *for all* $\sigma \in S \setminus A$, $\hat{q}_\sigma = \frac{a_\sigma + b_\sigma}{2}$ *for all* $\sigma \in A \setminus \{\eta\}$, *and* $\hat{q}_\eta = 1 - \sum_{\sigma \in S \setminus \{\eta\}} \hat{q}_\sigma$.

(iii) *If* $\alpha < 1/2$, *then* $d_\sigma < c_\sigma$ *for all* $\sigma \in A \setminus \{\eta\}$ *and* U *is maxmax expected utility* (0*-MEU) with set of priors* $\widehat{\mathcal{C}} = \{q \in \mathcal{C} : q_\sigma \in [d_\sigma, c_\sigma]\}$.

Hence, to study the spectrum of preferences represented by the α-MEU model (2.1) beyond the maxmin, maxmax and SEU preferences we have to specify a suitable set of priors. One typical choice is to assume that the set of priors of α-MEU model is the one that describes the uncertainty (i.e. the risk and the ambiguity) of the framework under study. The set that describes the uncertainty of the extended Ellsberg framework is the one that contains all priors such that the probabilities on the risky states equal the known probabilities π_R, $R \in S \setminus A$. We call this set $\mathcal{C}_{\max}$ and denote this class of models by α-$\mathcal{C}_{\max}$-MEU. Next section shows that indeed α-$\mathcal{C}_{\max}$-MEU utilities do reduce to maxmin, maxmax or SEU utilities. Moreover, the choice of $\mathcal{C}_{\max}$ as set of priors is suitable for the aim of this paper as it allows to interpret the parameter α as a measure of the agent's degree of ambiguity aversion and thus allows us to study the α-MEU agent's portfolio choice as a function of the different degree of agent's risk aversion; see Section 4.2.[25]

4.1 The α-$\mathcal{C}_{\max}$-MEU model

The α-MEU utility in (2.1) with $\mathcal{C} = \mathcal{C}_{\max}$ can be rewritten as

$$U(w) = \sum_{R \in S \setminus A} \pi_R\, u(w_R) + \Big(1 - \sum_{R \in S \setminus A} \pi_R\Big) \left[\alpha\, u(w^A_{\min}) + (1-\alpha)\, u(w^A_{\max})\right] \tag{4.2}$$

[25]The interpretation of α as a measure of ambiguity can be lost if the set of priors $\mathcal{C}$ is smaller than the set of priors $\mathcal{C}_{\max}$ that describes the uncertainty of the setting. A $\mathcal{C}$ strictly smaller than $\mathcal{C}_{\max}$ can reflect both additional information and less aversion towards ambiguity. However, in general the intuitive interpretation of α as an ambiguity aversion parameter is not warranted. One of the reasons is the potential multiplicity of representations of preferences as either α-MEU or maxmin/maxmax; see Section 3. The underlying subtle question is linked to the precise notion of the ambiguity in a problem, which has been debated in the decision theory literature; see, e.g., Siniscalchi (2006), Ghirardato et al. (2004), and Machina and Siniscalchi (2014).

where $w^A_{\min}$ and $w^A_{\max}$ is respectively the smallest and the largest wealth in the portfolio $w \in \mathbb{R}^{m+l}$ allocated among the l ambiguous states, that is

$$(4.3) \qquad w^A_{\min} := \min_{\sigma \in A} w_\sigma, \quad \text{and} \quad w^A_{\max} := \max_{\sigma \in A} w_\sigma.$$

The α-$\mathcal{C}_{\max}$-MEU utility (4.2) shows that when $l \geq 3$ and $\alpha \in (0,1)$, α-MEU utilities do not reduce to maxmin, maxmax nor to SEU utilities.[26] For instance, while SEU and maxmin utilities are always concave if u is concave, the α-$\mathcal{C}_{\max}$-MEU utility is concave if and only if $\alpha = 1$ (for a proof see Appendix C).[27] Thus the equivalence result (Proposition 3.1) only holds with two ambiguous states. In Section 4.4 we show that the α-$\mathcal{C}_{\max}$-MEU model and the maxmin model also imply different portfolio choices. Thus, in contrast to the standard Ellsberg framework, in the extended Ellsberg framework ambiguity averse α-$\mathcal{C}_{\max}$-MEU and maxmin portfolio choices are not anymore observationally equivalent.

4.2 The α-$\mathcal{C}_{\max}$-MEU portfolio choice

Before stating the proposition that characterizes the portfolio choice of α-$\mathcal{C}_{\max}$-MEU agents we introduce few notations that we use hereafter. Given a pricing rule $p \in \mathbb{R}^{m+l}$, $p^A_{\min}$ denotes the lowest (minimum) price among the ambiguous state prices, that is

$$p^A_{\min} := \min_{\eta \in A} p_\eta$$

and I the set that contains all ambiguous states with lowest price, that is

$$I := \{\sigma \in A \mid p_\sigma = p^A_{\min}\}.$$

Finally, by $\tilde{\pi}$ we denote the prior which assigns to the risky states the corresponding known probabilities π_R, $R \in S \setminus A$, and to each ambiguous state equal probability $\tilde{\pi}_a := \frac{\sum_{R \in S \setminus A} \pi_R}{l}$.

[26]It easy to see that also when the set of priors $\mathcal{C}$ is a strict subset of $\mathcal{C}_{\max}$, the α-MEU utility cannot in general be rewritten neither as 1-MEU, 0-MEU nor SEU, and is not concave.

[27]Another way to see this is to observe that the α-$\mathcal{C}_{\max}$-MEU utility from a portfolio $w \in \mathbb{R}^{m+l}$ on the ambiguous states only depends on $w^A_{\min}$ and $w^A_{\max}$. This is not the case for a maxmin (maxmax) utility model, as long as the state space contains more than two ambiguous states. The utility of the maxmin model from a portfolio $w \in \mathbb{R}^{m+l}$ will be a function of the portfolio's smallest wealth $w^A_{\min}$ (respectively, the portfolio's largest wealth $w^A_{\max}$) and then, depending on the set of priors, of the second smallest wealth (respectively, the second largest wealth) and so on, until the sum of the probabilities of the states in which these wealths are allocated reaches $(1 - \sum_{R \in S \setminus A} \pi_R)$.

Proposition 4.2. *Suppose that the state price vector p satisfies $p_\sigma > 0$ for all $\sigma \in S$. Consider an α-$\mathcal{C}_{\max}$-MEU agent. Let $\alpha \in (0,1]$ and suppose there is an optimal portfolio.*

- *If*

$$\alpha < 1 - \frac{p^A_{\min}}{1 - \sum_{R \in S \setminus A} p_R} = 1 - \frac{p^A_{\min}}{\sum_{\nu \in A} p_\nu} \tag{4.4}$$

  there are $|I|$ optimal portfolios: all optimal portfolios coincide on the risky states whereas on the ambiguous states they only take two different values $\overline{w} \in \mathbb{R}$ and $\underline{w} \in \mathbb{R}$ with $\underline{w} < \overline{w}$, which are the same for all optimal portfolios. Every optimal portfolio is obtained by choosing a single ambiguous state $\nu \in I$ ($p_\nu = p^A_{\min}$) among the cheapest ones and then setting

$$\begin{cases} w_\nu = \overline{w} \\ w_\eta = \underline{w} \text{ for the remaining } (l-1) \text{ ambiguous states } \eta \in A \setminus \{\nu\}. \end{cases} \tag{4.5}$$

  Hence, for all optimal portfolios $w^A_{\min} = \underline{w}$ and $w^A_{\max} = \overline{w}$.

- *If*

$$\alpha \geq 1 - \frac{p^A_{\min}}{\sum_{\nu \in A} p_\nu} \quad \text{or equivalently} \quad \alpha \geq 1 - \frac{p_\eta}{\sum_{\nu \in A} p_\nu}, \forall \eta \in A, \tag{4.6}$$

  the optimal portfolio w is unique and unambiguous, i.e. $w^A_{\max} = w^A_{\min}$. In particular, when $\alpha = 1$, the optimal portfolio w is always unique and unambiguous.

If $\alpha = 0$, there is no optimal portfolio.

Corollary 4.3. *The α-$\mathcal{C}_{\max}$-MEU model implies portfolio inertia both at the unambiguous and at the ambiguous portfolio* (4.5).

The proof of Proposition 4.2 is provided in Appendix D. The arguments used in the proof show that Proposition 4.2 holds true for any $\alpha \in (0,1]$ also when the utility u in (2.1) has a bounded domain, as long as the set of feasible portfolios remains convex and the utility is differentiable.[28] The only difference that a utility with bounded domain would bring is the existence of optimal

[28] These properties are used in the proofs of Lemmas D.1–D.5 that in turns prove Proposition 4.2, and can be ensured for instance by requiring that the feasible portfolios are in the interior of the utility domain.

portfolios of the 0-MEU agent.[29]

The proof of Corollary 4.3 is straightfoward. For simplicity, suppose $m = 0$. Since the optimal portfolio choice depends only on α and $p^A_{\min}$ (see (4.4) and (4.6)), the optimal portfolio remains optimal whenever the state price vector changes but the ambiguous state with price $p^A_{\min}$ and the price $p^A_{\min}$ remain the same.

The following example illustrates Proposition 4.2 when the state space S contains $m = 1$ risky state and $l = 3$ ambiguous states. Note that $l = 3$ yields $p^A_{\min} \leq \frac{1-p_R}{3}$ which is equivalent to $1 - \frac{p^A_{\min}}{1-p_R} \geq 2/3$. From the last inequality and Condition (4.4), it follows that any agent with $\alpha \in (0, 2/3)$ prefers an ambiguous portfolio[30].

Example 4.4. *Let $S = \{R\} \cup A$ where $A = \{G, B, Y\}$. Consider an α-$\mathcal{C}_{\max}$-MEU agent and let $w = (w_R, w_G, w_B, w_Y) \in \mathbb{R}^4$ be her optimal portfolio. Without loss of generality, let $0 < p_G \leq p_B \leq p_Y$.*

- *Let $\alpha \in (0, 2/3)$. Then the optimal portfolio is always exposed to ambiguity. In particular there are $w_R, \overline{w}, \underline{w} \in \mathbb{R}$ with $\overline{w} > \underline{w}$ such that:*
  - (i) *if $p_G < p_B$ ($p^A_{\min} = p_G$ and $I = \{G\}$), the optimal portfolio is unique and reads $w = (w_R, \overline{w}, \underline{w}, \underline{w})$*
  - (ii) *if $p_G = p_B < p_Y$ ($p^A_{\min} = p_G = p_B$ and $I = \{G, B\}$), then there are two optimal portfolios, namely $(w_R, \overline{w}, \underline{w}, \underline{w})$ and $(w_R, \underline{w}, \overline{w}, \underline{w})$*
  - (iii) *if $p_G = p_B = p_Y$ ($p^A_{\min} = p_G = p_B = p_Y = \frac{1-p_R}{3}$ and $I = A$), then there are three optimal portfolios: $(w_R, \overline{w}, \underline{w}, \underline{w})$, $(w_R, \underline{w}, \overline{w}, \underline{w})$, and $(w_R, \underline{w}, \underline{w}, \overline{w})$.*
- *Let $\alpha \in [\frac{2}{3}, 1]$. The optimal portfolio w is unambiguous (i.e. $w_G = w_B = w_Y$) if and only if $\alpha \geq 1 - \frac{p^A_{\min}}{1-p_R}$ or equivalently $\alpha \geq 1 - \frac{p_\eta}{1-p_R}, \forall \eta \in A = \{G, B, Y\}$. This is always the case if $p_G = p_B = p_Y$ or if $\alpha = 1$. Otherwise, i.e. if $\alpha < 1 - \frac{p^A_{\min}}{1-p_R}$, either* (i) or (ii) holds.

Proposition 4.2 and Example 4.4 show that an α-$\mathcal{C}_{\max}$-MEU agent with $\alpha \in (0, 1)$, facing a state price vector $p \in \mathbb{R}^{m+l}$, optimally chooses only two type of portfolios: either an unambiguous portfolio (with no exposure to ambiguity, allocating equal wealth in each ambiguous state), or an

[29] The non-existence of the optimal portfolio of 0-MEU agent is due to the fact that when the utility is defined on the whole real line, the agent can go arbitrarily long in one of the ambiguous states and still satisfy the budget constraint by going arbitrarily short in another ambiguous state. A utility with bounded domain would imply the existence of an optimal portfolio for the 0-MEU agent as the bounded domain will prevent the agent from going arbitrarily short; see Lemma D.5.

[30] The dependence of the portfolio choice on the number of ambiguous states l is discussed in Section 4.2.3.

ambiguous portfolio with the specific exposure to ambiguity in (4.5), i.e. allocating more wealth $\overline{w}$ to one of the cheapest ambiguous state and less equal wealth $\underline{w}$ to each of the remaining $(l-1)$ ambiguous states. This portfolio can be seen as an unambiguous portfolio with equal wealth $\underline{w}$ in each ambiguous state, plus a bet of $(\overline{w}-\underline{w})>0$ on one of the cheapest state. The larger the difference $\overline{w}-\underline{w}$, the more the portfolio is exposed to ambiguity.

The choice between unambiguous and ambiguous portfolios only depends on α and the ratio of the lowest price among the ambiguous state prices $p^A_{\min}$, to the total sum of the ambiguous state prices, $\sum_{\eta\in A} p_\eta$. If (4.6) holds, the optimal portfolio is unambiguous and unique. Otherwise if (4.4) holds, the optimal portfolio is ambiguous. The ambiguous portfolio, when optimal, is unique if there is only one ambiguous state with price $p^A_{\min}$, i.e. $|I|=1$. If the ambiguous states with price $p^A_{\min}$ are more than one, i.e. $|I|>1$, then there are $|I|$ optimal ambiguous portfolios. The α-$\mathcal{C}_{\max}$-MEU agent finds equally optimal to bet $\overline{w}-\underline{w}$ on any of the $|I|$ cheapest ambiguous states, because ambiguous states with equal prices are indistinguishable from an informational point of view. All optimal portfolios provide the same exposure to ambiguity. For an illustration, see Example 4.4.

In the following we discuss the dependence of the α-$\mathcal{C}_{\max}$-MEU agent's optimal portfolio on the ambiguity aversion, risk aversion and number of ambiguous states.

4.2.1 Impact of ambiguity aversion on portfolio choice

To understand how the ambiguity aversion parameter α determines the optimal exposure to ambiguity, we rewrite the α-$\mathcal{C}_{\max}$-MEU utility (4.2) from portfolio $w\in\mathbb{R}^{m+l}$ as

$$U(w)=\sum_{R\in S\setminus A}\pi_R u(w_R)+(1-\sum_{R\in S\setminus A}\pi_R)\left[u(w^A_{\min})+(1-\alpha)(u(w^A_{\max})-u(w^A_{\min}))\right]. \tag{4.7}$$

This equation shows that the coefficient $(1-\alpha)$ weights the utility $(u(w^A_{\max})-u(w^A_{\min}))$ that the α-$\mathcal{C}_{\max}$-MEU agent derives from the maximal exposure to ambiguity of the portfolio w, that is from $w^A_{\max}-w^A_{\min}$. When α increases the utility from the exposure to ambiguity decreases: when $\alpha=0$ this utility is highest, when $\alpha=1$ the utility is zero. This implies that, the higher is α, the smaller is the exposure to ambiguity of the agent's optimal portfolio.

We now study how the parameter α and the utility u determine the optimal allocation of wealth to risky and ambiguous states. Note that the optimal allocation of wealth among the risky states only depend on the utility u; see (4.2).

Let R denote the risky state, and set $m=1$ for simplicity. The following holds true.

- *An increase of α decreases the demand for the ambiguous portfolio.*

  Inequalities (4.6) show that an increase of α decreases the set of prices for which an α-$\mathcal{C}_{\max}$-MEU agent prefers an ambiguous portfolio. The limit case is $\alpha = 1$ in which the optimal portfolio is always unambiguous.

- *An increase of α decreases the exposure to ambiguity of the ambiguous portfolio.*

  This can be deduced from the first order conditions satisfied by the optimal ambiguous portfolio (see (D.2) in Lemma D.4 when $m = 1$)

$$\frac{u'(\underline{w})}{u'(\overline{w})} = \frac{\sum_{\nu\in A\setminus\{\sigma\}} p_\nu}{p_\sigma}\frac{(1-\alpha)}{\alpha} \tag{4.8}$$
$$\frac{u'(w_R)}{u'(\overline{w})} = \frac{(1-\alpha)(1-\pi_R)p_R}{p_\sigma \pi_R} \quad \text{and} \quad \frac{u'(w_R)}{u'(\underline{w})} = \frac{\alpha(1-\pi_R)p_R}{\sum_{\nu\in A\setminus\{\sigma\}} p_\nu \pi_R}$$

  where σ denotes (one of) the cheapest state among the ambiguous states, i.e. $\sigma \in I$ and $p_\sigma = p^A_{\min}$, $w_\sigma = \overline{w}$ and $w_\eta = \underline{w}$ for all $\eta \in A \setminus \{\sigma\}$.

  The closer α is to 0, the larger is $\overline{w}-\underline{w}$.[31] When α increases, the exposure to ambiguity of the agent's optimal portfolio $\overline{w}-\underline{w}$ decreases (as the ratio $\frac{(1-\alpha)}{\alpha}$ decreases). When $\alpha \uparrow 1-\frac{p^A_{\min}}{\sum_{\nu\in A} p_\nu}$, the bet $\overline{w}-\underline{w} \downarrow 0$, that is the ambiguous portfolio becomes unambiguous; see Condition (4.6).[32]

- *When the ambiguous portfolio is optimal, an increase of α always leads to an increase in the risk premium for the cheapest ambiguous state.*

  An α-$\mathcal{C}_{\max}$-MEU agent chooses an ambiguous portfolio if and only if among the ambiguous states there is at least one state σ (one of the chepaest ambiguous states) that satifies $p_\sigma < (1-\alpha)(1-p_R)$; see (4.4). Hence, the larger is α, the smaller p_σ must be in order to make the agent choose an ambiguous portfolio.

Equalities (4.8) also show how α impacts the allocation of wealth between the risky state R and the ambiguous states. An increase of α decreases the difference $\overline{w} - w_R$. When $\alpha \uparrow 1 - \frac{p^A_{\min}}{\sum_{\nu\in A} p_\nu}$ the optimal portfolio tends to the unambiguous portfolio, and the optimal allocation of wealth between the risky and the ambiguous states is the same as that of an SEU with the prior $\tilde{\pi}$.

Figure 3 illustrates the impact of the parameter α on the α-$\mathcal{C}_{\max}$-MEU agent's optimal portfolio when the agent's utility u in (4.2) is a CARA utility when $m = 1$ and $l = 4$.

[31] In the limit, when $\alpha \to 0$, $\overline{w} - \underline{w} \to \infty$, and thus there is no optimum; see discussion after Proposition 4.2.

[32] When $\alpha \uparrow 1 - \frac{p^A_{\min}}{\sum_{\nu\in A} p_\nu}$, $1-\alpha \downarrow \frac{p^A_{\min}}{\sum_{\nu\in A} p_\nu}$ and thus $\frac{u'(\underline{w})}{u'(\overline{w})} \downarrow 1$.

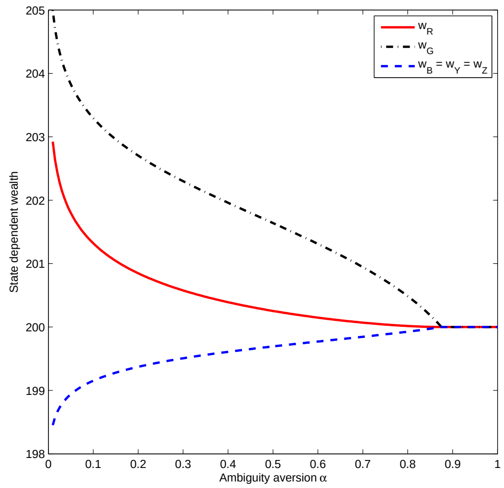


Figure 3: Optimal state dependent wealth α-$\mathcal{C}_{\max}$-MEU agent as a function of the degree of ambiguity aversion α, when the number of risk states $m = 1$, the number of ambiguous states $l = 4$, R is the risky state, and G, B, Y, Z are the ambiguous states. State prices are $p_R = 0.2$, $p_G = 0.1$, and p_B, p_Y, p_Z such that $p_B + p_Y + p_Z = 1 - p_R - p_G$ and $p_G = \min_{\nu \in \{G,B,Y,Z\}} p_\nu$. The agent's utility u in (4.2) is a CARA utility, $u(z) = 1 - e^{-\delta z}/\delta$, where $\delta = 1$.

4.2.2 Impact of risk aversion on portfolio choice

The utility function u in (4.2) that characterizes the risk aversion of the α-$\mathcal{C}_{\max}$-MEU agent also affects the extent to which the ambiguous portfolio in (4.5) is exposed to ambiguity. Equalities (4.8) show that, given a state price vector p and $\alpha \in (0,1)$, the more the utility function u is concave (i.e. the faster u' decreases) the smaller is the portfolio exposure $\overline{w} - \underline{w}$ to ambiguity and the difference $w_R - \overline{w}$. The dependence of the ambiguity exposure on risk aversion is illustrated in Figure 4 assuming CARA utility. When risk aversion increases, the agent eventually invests in the risk free asset.

4.2.3 Portfolio choice and number of ambiguous states

We now show that an α-$\mathcal{C}_{\max}$-MEU agent with $\alpha \in (0, \frac{l-1}{l})$ always prefers the ambiguous portfolio (4.5) and never chooses an unambiguous portfolio. The reason is that in a complete finite state space model when $\alpha \in (0, \frac{l-1}{l})$, Condition (4.4) is automatically satisfied. The normalization $\sum_{\eta \in A} p_\eta + \sum_{S \setminus A} p_R = 1$ yields

$$(4.9) \qquad p^A_{\min} \leq \frac{\sum_{\eta \in A} p_\eta}{l} \quad \text{and} \quad p^A_{\min} = \frac{\sum_{\eta \in A} p_\eta}{l} \Leftrightarrow \; p_\nu = p_\eta \; \forall \nu, \eta \in A.$$

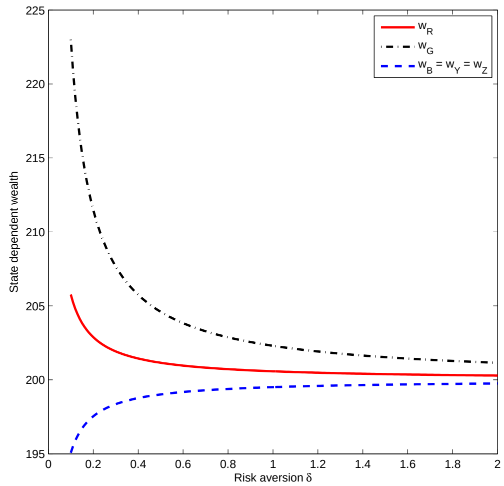


Figure 4: Optimal state dependent wealth of the α-$\mathcal{C}_{\max}$-MEU agent as a function of the degree of risk aversion δ, when the ambiguity aversion coefficient $\alpha = 0.3$, the number of risk states $m = 1$, the number of ambiguous states $l = 4$, R is the risky state, and G, B, Y, Z are the ambiguous states. State prices are $p_R = 0.2$, $p_G = 0.1$, and p_B, p_Y, p_Z such that $p_B + p_Y + p_Z = 1 - p_R - p_G$ and $p_G = \min_{\nu \in \{G,B,Y,Z\}} p_\nu$. The agent's utility u in (4.2) is a CARA utility, $u(z) = 1 - e^{-\delta z}/\delta$.

As $p^A_{\min} \leq \frac{\sum_{\eta \in A} p_\eta}{l}$ is equivalent to $1 - \frac{p^A_{\min}}{\sum_{\eta \in A} p_\eta} \geq \frac{l-1}{l}$, any $\alpha \in (0, \frac{l-1}{l})$ satisfies Condition (4.4).

When ambiguous states have equal price, from the equality in (4.9) and Proposition 4.2 it follows that the optimal portfolio of any α-$\mathcal{C}_{\max}$-MEU agent with $\alpha \in [\frac{l-1}{l}, 1]$ is unique and unambiguous. We formalize these concepts in the following corollaries.

Corollary 4.5. *In the setting of Proposition 4.2, any α-$\mathcal{C}_{\max}$-MEU agent with $\alpha \in (0, \frac{l-1}{l})$ always chooses a portfolio exposed to ambiguity of the type described in* (4.5).

Corollary 4.6. *In the setting of Proposition 4.2, suppose that the prices of the ambiguous states are all equal, i.e. $p_\nu = p_\eta$ for all $\nu, \eta \in A$.*

(i) *If $\alpha \in (0, \frac{l-1}{l})$, the optimal portfolios are the ambiguous portfolios in* (4.5). *Since $|I| = |A| = l$, the number of optimal portfolios equals the number of ambiguous states.*

(ii) *If $\alpha \in [\frac{l-1}{l}, 1]$, the optimal portfolio is unique and unambiguous, i.e. $w^A_{\max} = w^A_{\min}$.*

An increase of the number l of ambiguous states increases the interval of α-values $(0, \frac{l-1}{l})$ for which the corresponding α-$\mathcal{C}_{\max}$-MEU agent always chooses a portfolio exposed to ambiguity. The reason is that increasing l lowers the upper bound of $p^A_{\min}$ in (4.9) and consequently increases the interval of α-values that satisfy (4.4). For example, setting $m = 0$ for simplicity, when $l = 4$,

$p^A_{\min} \leq 0.25$ and the interval of α-values for which the α-$\mathcal{C}_{\max}$-MEU agent prefers exposure to ambiguity is $(0, 0.75)$. While when $l = 20$, $p^A_{\min} \leq 0.05$ and the interval of α-values is $(0, 0.95)$.

In Sections 4.3 and 4.4 we show how these results can be used in laboratory experiments to test the α-MEU model, measure the agent's degree of ambiguity aversion and distinguish between α-$\mathcal{C}_{\max}$-MEU agents and maxmin agents.

4.3 Attitudes towards ambiguity of the α-$\mathcal{C}_{\max}$-MEU model

To understand the different attitudes towards ambiguity expressed by the α-$\mathcal{C}_{\max}$-MEU model we start with the optimal portfolio choice when the prices of the ambiguous states are all equal. In this setting all the ambiguous states are equivalent from an informational point of view, and thus indistinguishable.[33] From Corollary (4.6) we know that when $l \geq 3$ and ambiguous state prices are equal:

- any α-$\mathcal{C}_{\max}$-MEU agent with $\alpha \in (0, \frac{l-1}{l})$ optimally chooses an ambiguous portfolio allocating equal wealth $\underline{w}$ on each ambiguous state plus a bet of size $\overline{w} - \underline{w} > 0$ made indifferently on one of the l ambiguous states. The number of optimal portfolios is equal to the number l of ambiguous states.

- any α-$\mathcal{C}_{\max}$-MEU agent with $\alpha \in [\frac{l-1}{l}, 1]$ optimally chooses an unambiguous portfolio with equal wealth in each ambiguous state. The optimal portfolio is unique.

The fact that α-$\mathcal{C}_{\max}$-MEU agents with $\alpha < \frac{l-1}{l}$ makes a bet on one among the l ambiguous states despite these states are all indistinguishable shows an ambiguity loving (or seeking) behavior of these agents. The ambiguity seeking behavior becomes more pronounced when α decreases. Indeed, as shown in Section 4.2.1, the smaller is α the larger is the exposure to ambiguity $\overline{w} - \underline{w}$ in the agent's optimal portfolio.

In contrast, α-$\mathcal{C}_{\max}$-MEU agents, with $\alpha \geq \frac{l-1}{l}$, do not show any ambiguity seeking behavior. When facing indistinguishable ambiguous states they optimally choose an unambiguous portfolio with equal wealth on each ambiguous state.

We observe that when $\alpha = \frac{l-1}{l}$, the α-$\mathcal{C}_{\max}$-MEU agent is not equivalent, not even observationally, to an ambiguity neutral SEU agent.[34] The $\frac{l-1}{l}$-$\mathcal{C}_{\max}$-MEU agent and the SEU agent with prior

[33] A comparatively high price in one of the ambiguous state may make the agents believe that this state has a higher probability of occurrence than the other ambiguous states, even though in the Ellsberg framework an exact knowledge of the probabilities is not available and the ambiguous states are "equally ambiguous".

[34] In Standard Ellsberg framework, when $l = 2$, any α-MEU utility with $\alpha = \frac{l-1}{l} = \frac{1}{2}$ reduces to a SEU utily; see Proposition 3.1.

$\tilde{\pi}$ choose the same unambiguous portfolio when ambiguous state prices are equal. However, the two agents choose different portfolios when the ambiguous states with cheapest price are more than one and less than $l-1$ (i.e., $1 <| I |< l-1$).[35] For example, when $m=0$, $l=3$, and two cheapest ambiguous states, G and B, there are two $\frac{2}{3}$-$\mathcal{C}_{\max}$-MEU optimal portfolios: $(w_G, w_B, w_Y) = (\overline{w}, \underline{w}, \underline{w})$ and $(w_G, w_B, w_Y) = (\underline{w}, \overline{w}, \underline{w})$, for some $\overline{w} > \underline{w}$. While the SEU optimal portfolio is unique and equals $(y_G, y_B, y_Y) = (\overline{y}, \overline{y}, \underline{y})$ for some $\overline{y} > \underline{y}$.

4.4 Disentangling between α-$\mathcal{C}_{\max}$-MEU and maxmin agents

The utility of a maxmin agent from some state dependent wealth $w \in \mathbb{R}^{l+m}$ is

$$U(w) = \sum_{R \in S \setminus A} \pi_R \, u(w_R) + \min_{\pi \in \mathcal{C}} \sum_{\sigma \in A} \pi_\sigma \, u(w_\sigma) \tag{4.10}$$

where $\mathcal{C} \subseteq \mathcal{C}_{\max}$ is a convex and closed set of priors. When $\mathcal{C} = \mathcal{C}_{\max}$ this utility is equal to the α-$\mathcal{C}_{\max}$-MEU utility (4.2) when $\alpha = 1$, and provides the maximal degree of ambiguity aversion. Shrinking the set of priors $\mathcal{C}$ in (4.10) decreases the exposure to ambiguity of the maxmin optimal portfolio, like decreasing the parameter α in (4.2) decreases the exposure to ambiguity of the α-$\mathcal{C}_{\max}$-MEU optimal portfolio.

The equivalence result in Proposition 3.1 shows that in the standard Ellsberg framework (two ambiguous states, i.e., $|A| = l = 2$) α-MEU utilities with $\alpha > \frac{1}{2}$ are maxmin utilities, and thus α-MEU preferences cannot be distinguished from maxmin preferences. In the following we show that in an extended Ellsberg framework (three or more ambiguous states, i.e., $|A| = l \geq 3$) this distinction can instead be achieved.[36] To disentangle α-MEU from maxmin preferences we exploit the α-$\mathcal{C}_{\max}$-MEU portfolio choice derived in Section 4.2 and the following lemma.

Lemma 4.7. *Suppose that $p_\sigma = p_\eta$ for all $\sigma, \eta \in A$. Then any maxmin agent* (4.10) *with a set of priors $\mathcal{C}$ such that $\tilde{\pi} \in \mathcal{C}$ takes an unambiguous portfolio.*

This lemma shows that when facing ambiguous states with equal prices, *any* maxmin agent with a set of priors $\mathcal{C}$ that includes $\tilde{\pi}$ chooses a portfolio with no exposure to ambiguity. In Ellsberg frameworks where the ambiguous states are all "equally ambiguous" (i.e., indistinguishable from

[35] In fact, when $1 <| I |< l-1$, the optimal portfolio of any α-$\mathcal{C}_{\max}$-MEU agent with $\alpha \in [0,1]$ is different from the SEU optimal portfolio.

[36] In Section 4.2 we observe that the equivalence result does not hold when the number of ambiguity states is larger than two. However, the fact that α-MEU utilities cannot be rewritten as 1-MEU utilities does not imply that their portfolio choice may not be observationally equivalent.

a probabilistic point of view) it is natural that the prior $\tilde{\pi}$ belongs to the maxmin agent's set of priors.[37]

In the following we point out the differences between the α-MEU optimal portfolio and the maxmin optimal portfolio that may be used in a multiple-stage laboratory experiment to disentangle ambiguity seeking from non-ambiguity seeking agents, and among the latter, α-$\mathcal{C}_{\max}$-MEU from maxmin agents.

4.4.1 Ambiguity seeking and non-ambiguity seeking agents

The identification of ambiguity seeking α-$\mathcal{C}_{\max}$-MEU agents, $\alpha \in (0, \frac{l-1}{l})$, and non-ambiguity seeking agents, α-$\mathcal{C}_{\max}$-MEU agents with $\alpha \in [\frac{l-1}{l}, 1)$ and the maxmin agents, can be achieved by observing their different portfolio choices. For example, when $m = 1$, $l = 3$, and the ambiguous states have equal prices, ambiguity seeking α-$\mathcal{C}_{\max}$-MEU agents ($\alpha < \frac{2}{3}$) should choose and be indifferent among the following three portfolios: $(w_R, \overline{w}, \underline{w}, \underline{w})$, $(w_R, \underline{w}, \overline{w}, \underline{w})$ and $(w_R, \underline{w}, \underline{w}, \overline{w})$ for some $\overline{w} > \underline{w}$. While α-$\mathcal{C}_{\max}$-MEU agents with $\alpha \geq \frac{2}{3}$ and maxmin agents should choose one (unique) unambiguous portfolio.

4.4.2 Non-ambiguity seeking and maxmin agents

Once the distinction between agents with and without ambiguity seeking attitudes is achieved, additional experiments involving only the agents with non-ambiguity seeking attitude can be carried out to disentangle maxmin agents from α-$\mathcal{C}_{\max}$-MEU agents with $\alpha \in [\frac{l-1}{l}, 1)$. This distinction can be achieved observing that the maxmin optimal portfolio is typically unique, while the α-$\mathcal{C}_{\max}$-MEU ambiguous optimal portfolios are not unique when there are more than one ambiguous states with cheapest price and less than $l - 1$, i.e., $1 <| I |< l - 1$.[38] For example, when $m = 1$, $l = 3$, and the state price vector $p \in \mathbb{R}^{1+3}$ is such that $p^A_{\min} = p_G = p_B < p_Y$, any α-$\mathcal{C}_{\max}$-MEU agent with $\alpha \in [\frac{l-1}{l}, 1 - \frac{p_G}{1-p_R}] = [\frac{2}{3}, 1 - \frac{p_G}{1-p_R}]$ chooses and is indifferent between the two portfolios $(w_R, \overline{w}, \underline{w}, \underline{w})$ and $(w_R, \underline{w}, \overline{w}, \underline{w})$.[39] Thus, for instance by asking the agents in a sequence of experimental sections to choose their optimal portfolios without changing the prices, we expect to see the α-$\mathcal{C}_{\max}$-MEU agent switching its choice between the two optimal portfolios, while the maxmin agent chooses the

[37]If a set of priors $\mathcal{C}$ is symmetric (i.e., permutation invariant) in the ambiguous coordinates then $\tilde{\pi} \in \mathcal{C}$. Thus, if $\tilde{\pi} \notin \mathcal{C}$ some ambiguous states will be systematically overweighted and other underweighted. Note that $\mathcal{C}_{\max}$ includes $\tilde{\pi}$ and is symmetric in the ambiguous states.

[38]The optimal portfolio of a maxmin agent is typically unique. In particular this is always the case when the maxmin utility is strictly concave.

[39]When $p_B = p_G$ approach zero the right-hand side of the interval $[\frac{2}{3}, 1 - \frac{p_G}{1-p_R}]$ approaches 1.

same portfolio in any section.

The distinction between α-$\mathcal{C}_{\max}$-MEU agents with $\alpha \in [\frac{l-1}{l}, 1)$ and the maxmin agents cannot be achieved via the observation of one single portfolio choice. The reason is that depending on her set of priors a maxmin agent may also optimally choose one of the two portfolios which are optimal for the α-$\mathcal{C}_{\max}$-MEU agent with $\alpha \in [\frac{2}{3}, 1 - \frac{p_G}{1-p_R}]$, i.e., either the portfolio $(w_R, \overline{w}, \underline{w}, \underline{w})$ or $(w_R, \underline{w}, \overline{w}, \underline{w})$. Indeed, one can show that for any given portfolio there exists a set of priors $\mathcal{C}$ for which the associated maxmin agent chooses that portfolio as optimal.

5 Ambiguity seeking behaviors and market equilibrium

In this section we consider a simple market populated by SEU and ambiguity sensitivity agents and show that the existence of the market equilibrium depends on whether ambiguity seeking agents are or not present in the market. In fact, as we show in the following, the ambiguity seeking α-$\mathcal{C}_{\max}$-MEU agents may prevent the existence of the equilibrium that otherwise exists if, together with the SEU agents, the non-ambiguity seeking α-$\mathcal{C}_{\max}$-MEU or maxmin agents populate the market.

Example 5.1. *Consider a market with two agents and suppose that the state dependent total endowment $W \in \mathbb{R}^{m+l}$ is such that $W_\eta = W_\nu$, for all $\eta, \nu \in A$.*

(i) *First suppose that in the market there is a SEU agent with prior $\tilde{\pi}$ and a non-ambiguity seeking agent, i.e. either an α-$\mathcal{C}_{\max}$-MEU agent with $\alpha \in [\frac{l-1}{l}, 1)$, or a maxmin agent with $\tilde{\pi} \in \mathcal{C}$.*[40] *It is easy to see that in this market the equilibrium exists. Specifically, the state price vector in equilibrium is $p \in \mathbb{R}^{l+m}$ with $p_\eta = \frac{1-\sum_{S\setminus A} p_R}{l}$, $\forall \eta \in A$; the SEU optimal portfolio is $y \in \mathbb{R}^{m+l}$ with $y_\eta = y_\nu$, $\forall \eta, \nu \in A$, and the ambiguity averse agent's optimal portfolio is $w \in \mathbb{R}^{m+l}$ with $w_\eta = w_\nu$, $\forall \eta, \nu \in A$, where y and w are such that $y_\eta + w_\eta = W_\eta$, $\forall \eta \in A$.*[41]

(ii) *Now suppose that in the market, together with the SEU agent, there is an ambiguity seeking α-$\mathcal{C}_{\max}$-MEU agent, i.e. $\alpha \in (0, \frac{l-1}{l})$. This agent only chooses ambiguous portfolio of the type* (4.5) *(see Corollary 4.5), that is $w \in \mathbb{R}^{l+1}$ such that $w_\sigma = \overline{w}$, $w_\eta = \underline{w}$, $\forall \eta \in A \setminus \{\sigma\}$, $\overline{w} > \underline{w}$, where σ is (one of the) the cheapest ambiguous state, i.e. $p_\sigma \leq p_\eta$, $\forall \eta \in A \setminus \{\sigma\}$. If the equilibrium exists, to clear the market, the SEU optimal portfolio $y \in \mathbb{R}^{l+1}$ has to satisfy $y_\sigma = W_\sigma - \overline{w} < W_\eta - \underline{w} = y_\eta$, $y_\eta = W_\eta - \underline{w} = W_\nu - \underline{w} = y_\nu$, $\forall \eta, \nu \in A \setminus \{\sigma\}$, that is*

[40] A necessary condition for the existence of the equilibrium is that beliefs are consistent across agents in the market. We recall that $\tilde{\pi}$ is in the set of prior $\mathcal{C}_{\max}$ of the α-$\mathcal{C}_{\max}$-MEU.

[41] When the ambiguous states have equal prices, the unambiguous portfolio is optimal both for the α-$\mathcal{C}_{\max}$-MEU (see Corollary 4.6) and the maxmin agent (see Lemma 4.7).

$y_\sigma < y_\eta = y_\nu$, $\forall \eta, \nu \in A \setminus \{\sigma\}$. This portfolio will is optimal for the SEU agent only if (see (A.3)*) the equilibrium state prices satisfy $p_\sigma > p_\eta = p_\nu$, $\forall \eta, \nu \in A \setminus \{\sigma\}$, but this condition is in contradiction with the α-$\mathcal{C}_{\max}$-MEU portfolio optimality condition $p_\sigma \leq p_\eta$, $\forall \eta, \nu \in A \setminus \{\sigma\}$. This implies that there is no equilibrium for this market.*

To derive some economic intuition as to why the presence of ambiguity seeking α-$\mathcal{C}_{\max}$-MEU agents may prevent the existence of market equilibrium, note that in the α-$\mathcal{C}_{\max}$-MEU utility (4.2), $(1 - \sum_{R \in S \setminus A} \pi_R)(1 - \alpha)$ plays the role of the "fictitious" probability of the state σ on which the highest wealth $w^A_{\max}$ is allocated. The more ambiguity seeking is the agent (i.e., the smaller is α), the higher is the probability of the state σ, and (ceteris paribus) the higher should be the price of state σ in equilibrium. However, the portfolio optimality condition of ambiguity seeking agents requires that σ is one of cheapest ambiguous state in equilibrium. These potential contradicting conditions may prevent the existence of the equilibrium.

As an illustration of this point, we specify Example 5.1 (ii) to the CARA case, i.e., when the utility u of both the SEU and the ambiguity seeking α-$\mathcal{C}_{\max}$-MEU agents equal $u(z) = 1 - \frac{e^{-z}}{z}$. In this case, the existence of equilibrium requires the state prices to satisfy:

$$
\begin{aligned}
\frac{p_\sigma}{p_\eta} &= \left(\frac{(l-1)(1-\alpha)}{\alpha}\right)^{\frac{1}{2}}, \ \forall \eta \in A \setminus \{\sigma\} \\
\frac{p_\sigma}{p_\eta} &\leq 1, \ \forall \eta \in A \setminus \{\sigma\} \\
\frac{p_\eta}{p_\nu} &= 1, \ \forall \eta, \nu \in A \setminus \{\sigma\} \ \forall \eta \in A \setminus \{\sigma\}.
\end{aligned}
$$

Any value of $\alpha \in (0, \frac{l-1}{l})$ implies a ratio of probability of the state σ to probability of the state $\eta \in A \setminus \{\sigma\}$ strictly larger than 1, i.e., $\frac{(1-\alpha)(l-1)}{\alpha} > 1$, and thus $p_\sigma > p_\eta$, which is in contradiction with the α-$\mathcal{C}_{\max}$-MEU portfolio optimality condition $\frac{p_\sigma}{p_\eta} \leq 1, \forall \eta \in A \setminus \{\sigma\}$.[42]

For a non-ambiguity seeking α-$\mathcal{C}_{\max}$-MEU agent with $\alpha \in [\frac{l-1}{l}, 1)$, the probability $(1 - \sum_{R \in S \setminus A} \pi_R)(1 - \alpha)$ of the state on which $w^A_{\max}$ is allocated is bounded from above by the probability $\tilde{\pi}_a$, and decreases when α increases. This is also true in the maxmin model. Indeed a necessary condition for having a maxmin agent to choose the ambiguous portfolio in (4.5) is that the prior π^* that realizes

[42] The α-$\mathcal{C}_{\max}$-MEU utility assigns probability $(1 - \alpha)(1 - \sum_{R \in S \setminus A} \pi_R)$ to the state σ on which the highest wealth $\overline{w}$ is allocated, and probability $\frac{\alpha(1 - \sum_{R \in S \setminus A} \pi_R)}{(l-1)}$ to each of the remaining states $\eta \in A \setminus \{\sigma\}$. The SEU-prior $\tilde{\pi}$ does not apper in the inequalities characterizing the ambiguous state prices because $\tilde{\pi}$ assigns equal probability $\tilde{\pi}_a$ to each ambiguous state and thus cancels out. The total endowment W also cancels out because $W_\eta = W_\nu$, for all $\eta, \nu \in A$.

the minimum in (4.10) is bounded from above by the probability $\tilde{\pi}$.[43]

Because non-ambiguity seeking agents have bounded probability of the state on which the highest wealth is allocated, and their optimal portfolios may also be unambiguous facilitate the equilibrium prices to settle, as in Example 5.1.

6 Conclusion

The α-MEU model has been used in many theoretical and experimental studies to describe the behavior of agents under ambiguity. We show that in the standard Ellsberg framework (two ambiguous states) α-MEU preferences coincide with either maxmin, maxmax or subjective expected utility preferences, and derive equilibrium asset prices when the market is populated by ambiguity averse and subjective expected utility investors. Our theoretical results are strikingly in agreement with the laboratory experimental findings in Bossaerts et al. (2010), and show why ambiguity aversion does not wash out in equilibrium.

In an extended Ellsberg framework (three or more ambiguous states) we show that the α-MEU preferences do not coincide with maxmin, maxmax or subjective expected utility preferences and induce portfolio choices that are not observationally equivalent. We characterize the optimal portfolio choice of an α-$\mathcal{C}_{\max}$-MEU agent. This agent optimally chooses only between two types of portfolios: either an unambiguous portfolio, or an ambiguous portfolio with one specific exposure to ambiguity (that allocates more wealth to one of cheapest ambiguous states and less equal wealth to the other ambiguous states). The number of optimal ambiguous portfolios is equal to the number of ambiguous states with cheapest price. Our theoretical findings can inform laboratory experiments to disentangle between ambiguity seeking and non-ambiguity seeking agents, and among the latter, between α-$\mathcal{C}_{\max}$-MEU and maxmin agents. Finally, we find that when ambiguity seeking agents are present in the market they may prevent the existence of market equilibrium that otherwise would exist with non-ambiguity seeking agents.

[43] This can be shown by observing that $w_\sigma > w_\eta$, $\forall \eta \in A \setminus \{\sigma\}$ implies $u(w_\eta) - u(w_\sigma) < 0$, $\forall \eta \in A \setminus \{\sigma\}$. Then the optimal prior π^* has to be a prior which maximizes the sum of the probability of the state $\eta \in A \setminus \{\sigma\}$. Therefore, since $\tilde{\pi} \in \mathcal{C}$, then π^* is such that $\sum_{\eta \in A \setminus \{\sigma\}} \pi^*_\eta \geq \frac{1 - \sum_{S \setminus A} \pi_R}{l}(l-1)$ or equivalently $\pi^*_\sigma \leq \frac{1 - \sum_{S \setminus A} \pi_R}{l}$.

A Proof of Proposition 3.2

In the following, we briefly summarize how the interaction among SEU and maxmin agents impacts the equilibrium asset prices. This will provide us the tools to prove Proposition 3.2. Assume that $(p; w^1, \ldots, w^n)$ is an equilibrium with $p_\sigma > 0$ for all $\sigma \in \{R, G, B\}$. Then, the equilibrium price p satisfies

$$\lambda_n p \in \partial U^n(w^n) \tag{A.1}$$

for some $\lambda_n > 0$; see (F.3). Here $\partial U^n(w)$ denotes the supergradient of the criterion U^n of agent n at $w \in \mathbb{R}^3$. The supergradient of a SEU-agent with prior $\pi = (\pi_R, \pi_G, \pi_B)$ is simply the gradient

$$\partial U^n(w) = \{\big(\pi_R u'(w_R), \pi_G u'(w_G), \pi_B u'(w_B)\big)\}. \tag{A.2}$$

From (A.2) and the strict concavity of the utility function, it follows the well known fact that the optimal portfolio $w = (w_R, w_G, w_B)$ of a SEU agent is always such that the optimal choices of state dependent wealth are ranked opposite to the state-price/state-probability ratios, i.e.

$$w_\sigma > w_\nu \Leftrightarrow \frac{p_\sigma}{\pi_\sigma} < \frac{p_\nu}{\pi_\nu}, \quad \sigma, \nu \in \{R, B, G\}. \tag{A.3}$$

The supergradient of an agent with maxmin (1-MEU) preferences represented as in (3.3) is

$$\partial U^m(w) = \begin{cases} \{(\pi_R u'(w_R), c u'(w_G), (1 - \pi_R - c) u'(w_B))\} & \text{if } w_G > w_B \\ \{(\pi_R u'(w_R), d u'(w_G), (1 - \pi_R - d) u'(w_B))\} & \text{if } w_G < w_B \\ \{(\pi_R u'(w_R), (\lambda c + (1 - \lambda) d) u'(w_G), \\ (1 - \pi_R - (\lambda c + (1 - \lambda) d)) u'(w_B)) \mid \lambda \in [0, 1]\} & \text{if } w_G = w_B. \end{cases} \tag{A.4}$$

Using (A.1) and the shape of the supergradients we easily obtain the optimal portfolio choices that were already derived in Bossaerts et al. (2010). In particular, from (A.4) and the strict concavity of u it follows that

$$\begin{cases} w_G > w_B & \text{if and only if } \frac{p_G}{p_B} < \frac{c}{1 - \pi_R - c} \\ w_G < w_B & \text{if and only if } \frac{p_G}{p_B} > \frac{d}{1 - \pi_R - d} \\ w_G = w_B & \text{if and only if } \frac{p_G}{p_B} \in \left[\frac{c}{1 - \pi_R - c}, \frac{d}{1 - \pi_R - d}\right] \end{cases} \tag{A.5}$$

where $x/0 := \infty$. The larger the set of priors $\mathcal{C}$ in (3.4), the more likely a maxmin agent will take an unambiguous portfolio ($w_B = w_G$). In particular this will be always the case if $\mathcal{C} = \mathcal{C}_{\max} := \{(\pi_R, q, 1 - q - \pi_R) : q \in [0, 1 - \pi_R]\}$, because then the second respectively third coordinate of the supergradient in (A.4) will be 0 if either $w_G > w_B$ or $w_G < w_B$. Hence, $p_\sigma > 0$ for all $\sigma \in \{R, G, B\}$ and (A.1) imply that in equilibrium this agent will only take an unambiguous portfolios w. If $c > 0$ and/or $d < 1 - \pi_R$ in (3.4), then the multiple prior agent may also take an ambiguous portfolio in equilibrium. We observe that a maxmin agent holding an unambiguous optimal portfolio behaves as a SEU-agent who is not differentiating between the ambiguous states G and B, but merges them to an unambiguous state $\{G, B\}$ with probability $(1 - \pi_R)$. Indeed, from (A.4) and (A.1) it follows that

$$\text{(A.6)} \qquad \frac{p_{\{G,B\}}}{p_R} = \frac{(1-\pi_R)u'(w_{\{G,B\}})}{\pi_R u'(w_R)} \begin{cases} < \frac{(1-\pi_R)}{\pi_R} & \text{iff } w_{\{G,B\}} > w_R \\ > \frac{(1-\pi_R)}{\pi_R} & \text{iff } w_{\{G,B\}} < w_R \end{cases}$$

and thus

$$\text{(A.7)} \qquad \frac{p_{\{G,B\}}}{(1-\pi_R)} < \frac{p_R}{\pi_R} \quad \Leftrightarrow \quad w_{\{G,B\}} > w_R$$

$$\text{(A.8)} \qquad \frac{p_{\{G,B\}}}{(1-\pi_R)} > \frac{p_R}{\pi_R} \quad \Leftrightarrow \quad w_{\{G,B\}} < w_R \quad \text{(compare this to (A.3))},$$

where $p_{\{G,B\}} := p_G + p_B$ and $w_{\{G,B\}} := w_G = w_B$.

Proof of Proposition 3.2
Case 1: Let $W_R > W_G > W_B$. Since the 1-MEU agents take an unambiguous portfolio, the optimal portfolio of some SEU agent must satisfy $y_G > y_B$ which according to (A.3) is equivalent to

$$\text{(A.9)} \qquad \frac{p_B}{\pi_B} > \frac{p_G}{\pi_G}$$

which only leaves the ranking of p_R/π_R within (A.9) an open question. Suppose that the ranking of the ratios state-price/state-probability is as follows:

$$\text{(A.10)} \qquad \frac{p_R}{\pi_R} \geq \frac{p_B}{\pi_B} > \frac{p_G}{\pi_G}.$$

Then (A.3) implies that $y_G > y_B \geq y_R$ for any SEU agent, and rearranging (A.10) yields

$$\frac{p_G + p_B}{1-\pi_R} = \frac{p_G + p_B}{\pi_G + \pi_B} < \frac{p_R}{\pi_R}.$$

Consequently, according to (A.6), we must have for each 1-MEU agent that $w_R < w_G = w_B$. But this contradicts the clearing of the market and $W_R > W_G > W_B$. If the ranking is (3.6), then we have $y_G > y_R > y_B$ for each SEU agent according to (A.3). Denote by $y^\Sigma = (y_R^\Sigma, y_G^\Sigma, y_B^\Sigma)$ the sum over all optimal portfolios of the SEU agents and similarly by $w^\Sigma = (w_R^\Sigma, w_G^\Sigma, w_B^\Sigma)$ the sum over all optimal portfolios of the 1-MEU agents. The market clearing condition says $W_\sigma = y_\sigma^\Sigma + w_\sigma^\Sigma$ for every $\sigma \in \{R, G, B\}$. Since $y_G^\Sigma > y_R^\Sigma$ we conclude that

$$w_R^\Sigma = W_R - y_R^\Sigma > W_G - y_G^\Sigma = w_G^\Sigma.$$

Thus there must be at least one 1-MEU agent who's portfolio $w = (w_R, w_G, w_B)$ satisfies $w_R > w_G = w_B$ which implies that $(p_G + p_B)/p_R > (1-\pi_R)/\pi_R$ due to (A.6). But then, again by (A.6), we must have $w_R > w_G = w_B$ for all 1-MEU agents. In case of (3.7) (A.3) and (A.6) imply the claimed ranking of payoffs in the portfolios y and w.

Case 2: Let $W_G > W_R > W_B$. As in case one we conclude that $y_G > y_B$. Assume that the ranking of the ratio state-price/state-probability is as follows:

$$\text{(A.11)} \qquad \frac{p_R}{\pi_R} \geq \frac{p_B}{\pi_B} > \frac{p_G}{\pi_G}.$$

Then as in case 1 it follows that $y_G > y_B \geq y_R$ and $w_R < w_G = w_B$ which together with the clearing of the market contradicts $W_R > W_B$. Similarly it follows that the ranking

$$\frac{p_B}{\pi_B} > \frac{p_G}{\pi_G} \geq \frac{p_R}{\pi_R}$$

is not possible, since it would imply that $y_R \geq y_G > y_B$ and $w_R > w_G = w_B$ due to (A.6), again contradicting the assumed ranking of the aggregate wealth.

Case 3: Let $W_G > W_B > W_R$: Suppose that

$$\frac{p_B}{\pi_B} > \frac{p_G}{\pi_G} \geq \frac{p_R}{\pi_R}$$

then, $y_R \geq y_G > y_B$, and in view of (A.6) we obtain $w_R > w_G = w_B$ for every 1-MEU agent which again contradicts the market clearing and the assumed ranking $W_G > W_B > W_R$. Again (A.3), (A.6), and the clearing of the market imply the claimed ranking of payoffs in the portfolios y, w for the remaining possible rankings.

B Proof of Propositions 3.1 and 4.1

Proposition 3.1 is a special case of Proposition 4.1 since every set of priors in the standard Ellsberg framework is of the type $\mathcal{C}$ in (4.1), required in Proposition 4.1.

To prove Proposition 4.1 we observe that the maxmin utility with set of priors $\mathcal{C}$ can be written as

$$\text{(B.1)} \qquad u(w_\eta) + \sum_{\sigma \in S \setminus A} (u(w_\sigma) - u(w_\eta))\pi_\sigma + \sum_{\sigma \in A \setminus \{\eta\}} (u(w_\sigma) - u(w_\eta))^+ a_\sigma - (u(w_\sigma) - u(w_\eta))^- b_\sigma,$$

and the maxmax utility as

$$\text{(B.2)} \qquad u(w_\eta) + \sum_{\sigma \in S \setminus A} (u(w_\sigma) - u(\eta))\pi_\sigma + \sum_{\sigma \in A \setminus \{\eta\}} (u(w_\sigma) - u(w_\eta))^+ b_\sigma - (u(w_\sigma) - u(w_\eta))^- a_\sigma.$$

Consequently, the α-MEU utility is

$$U(w) = u(w_\eta) + \sum_{\sigma \in S \setminus A} (u(w_\sigma) - u(w_\eta))\pi_\sigma + \sum_{\sigma \in A \setminus \{\eta\}} (u(w_\sigma) - u(w_\eta))^+ c_\sigma - (u(w_\sigma) - u(w_\eta))^- d_\sigma.$$

where $c_\sigma := \alpha a_\sigma + (1-\alpha) b_\sigma$, and $d_\sigma := \alpha b_\sigma + (1-\alpha) a_\sigma$, $\sigma \in A \setminus \{\eta\}$. If $\alpha > 1/2$, then $c_\sigma < d_\sigma$; if $\alpha < 1/2$, then $d_\sigma < c_\sigma$; and finally $c_\sigma = d_\sigma$ for $\alpha = 1/2$. These facts, and comparing U for the different cases (i), (ii) and (iii) with (B.1) and (B.2) prove Proposition 4.1.

C Lack of concavity of the α-MEU utility

To see the lack of concavity of the α-$\mathcal{C}_{\max}$-MEU utility when $\alpha \neq 1$, consider portfolio w^1 such that $w^1_1 = 1, w^1_2 = 4$ and $w^1_j = 2, \forall j = 2, \ldots, l$, and portfolio w^2 such that $w^2_1 = 1, w^2_2 = 2, w^2_3 = 6$ and $w^2_j = 2, \forall j = 4, \ldots, l$. Let $w^\lambda = (w^\lambda_1, \ldots, w^\lambda_l)$ be their convex combination, i.e. $w^\lambda_j = \lambda w^1_j + (1-\lambda) w^2_j$, $j = 1, \ldots, l$, $\lambda \in [0,1]$. Take for instance $\lambda = 1/2$. Then $w^\lambda_1 = 1, w^\lambda_2 = 3, w^\lambda_3 =$

4 and $w_j^\lambda = 2, \forall j = 4, \dots, l$, and using (4.2)

$$\lambda U(w^1) + (1-\lambda)U(w^2) = \alpha u(1) + (1-\alpha)\frac{1}{2}(u(4) + u(6)) >$$
$$\alpha u(1) + (1-\alpha)u(4) = \alpha u(w_{\min}^{\lambda\, A}) + (1-\alpha)u(w_{\max}^{\lambda\, A}) = U(w^\lambda) = U((\lambda w^1 + (1-\lambda)w^2)).$$

D Proof of Proposition 4.2

Proposition 4.2 follows from Lemmas D.1–D.5 in the following.

Lemma D.1. *Suppose that the state price vector $p = (p_\sigma)_{\sigma \in S}$ satisfies $p_\sigma > 0$ for all $\sigma \in S$. Consider an α-$\mathcal{C}_{\max}$-MEU agent with $\alpha \in (0,1)$. Let $w = (w_\sigma)_{\sigma \in S} \in \mathbb{R}^n$ be an optimal portfolio for the α-$\mathcal{C}_{\max}$-MEU agent. Then, either w takes the same value on all ambiguous states, or there exist two disjoint subsets $\overline{A}$ and $\underline{A}$ of the set of ambiguous states A such that $\overline{A} \cup \underline{A} = A$ and two values $\overline{w}, \underline{w} \in \mathbb{R}$ such that $w_\sigma = \overline{w} > \underline{w} = w_\eta$ for all $\sigma \in \overline{A}$ and all $\eta \in \underline{A}$.*

Proof. Note that the only portfolio values on the ambiguous states on which the utility U in (4.2) depends are $w_{\max}^A$ and $w_{\min}^A$. We order the set of ambiguous states $A = \{\sigma_1, \dots, \sigma_l\}$ such that

$$w_{\sigma_1} \le w_{\sigma_2} \le \dots \le w_{\sigma_l}. \tag{D.1}$$

Let s be the number of strict inequalities in (D.1). Consider states $\nu_1, \dots \nu_{s+1} \in A$ such that $w_{\nu_1} < w_{\nu_2} < \dots < w_{\nu_{s+1}}$. Suppose there is a state $\eta \in A$ such that $w_\eta \neq w_{\max}^A$ and $w_\eta \neq w_{\min}^A$, namely suppose that $s \ge 2$. We now consider the function U in (4.2) as defined on $\mathbb{R}^{m+s+1}$, where we merge those ambiguous states in which w takes the same value. Let $\tilde{w} \in \mathbb{R}^{m+s+1}$ such that $\tilde{w}_R = w_R$ for all risky states $R \in S \setminus A$ and otherwise $\tilde{w}_{\sigma_i} = w_{\sigma_i}$ for $i = 1, \dots, s+1$. Then, $\tilde{w}$ is a maximizer for the function U restricted to the open set $C := \{x \in \mathbb{R}^{m+s+1} \mid x_{\sigma_1} < x_{\sigma_2} < \dots < x_{\sigma_{s+1}}\}$, which we call U_C, given the budget constraint $\tilde{p} \cdot \tilde{w} \le p \cdot e$. Here e is the initial portfolio and $\tilde{p} \in \mathbb{R}^{m+s+1}$ is obtained from p by summing up the prices of those states which are merged when forming $\tilde{w}$. As U_C is concave, according to (F.3), a multiple of $\tilde{p}$ is in the supergradient of U_C at $\tilde{w}$. However, this supergradient is equal to zero in any x_{σ_i}-direction, $i \in \{2, \dots, s\}$, because only the largest value and the smallest value on the ambiguous states matter for U. This contradicts the assumption $p_{\sigma_i} > 0$ for $i \in \{2, \dots, s\}$. □

Lemma D.2. *Assume Lemma D.1. If $p_\sigma < p_\eta$ for $\sigma, \eta \in A$, then the optimal portfolio w satisfies $w_\eta \le w_\sigma$.*

Proof. Suppose that the optimal portfolio w is such that $w_\eta > w_\sigma$. Let $\tilde{w}$ given by $\tilde{w}_\nu = w_\nu$ for all $\nu \in S \setminus \{\sigma, \eta\}$ and $\tilde{w}_\sigma = w_\eta$ and $\tilde{w}_\eta = w_\sigma$. Then $U(\tilde{w}) = U(w)$, but $p \cdot \tilde{w} < p \cdot w$ because $p \cdot (w - \tilde{w}) = (p_\eta - p_\sigma)(w_\eta - w_\sigma) > 0$. This contradicts the optimality of w, because increasing the wealth $\tilde{w}_\sigma$ one could achieve a strictly higher utility while still respecting the budget constraint. □

Lemma D.3. *Assume Lemma D.1. If the sets $\overline{A}$ and $\underline{A}$ associated to the optimal portfolio w are not empty, then $\overline{A} = \{\overline{\sigma}\}$ for a state $\overline{\sigma} \in I := \{\sigma \in A \mid p_\sigma = \min_{\eta \in A} p_\eta\}$. Moreover, any portfolio which equals w on the risky states and assigns the weight $w_{\max}^A$ to a single state in I and $w_{\min}^A$ to all the other ambiguous states is optimal. Hence, there are $|I|$ optimal portfolios.*

Proof. By contradiction suppose that there are two different states σ_1 and σ_2 in $\overline{A}$, i.e. that the optimal portfolio w is such that $w_{\sigma_1} = w_{\sigma_2} = w_{\max}^A$, and without loss of generality we assume that $p_{\sigma_1} \le p_{\sigma_2}$. Consider $\tilde{w}$ given by $\tilde{w}_\eta = w_\eta$ for all $\eta \in S \setminus \{\sigma_1, \sigma_2\}$ and $\tilde{w}_{\sigma_1} = 2w_{\max}^A - w_{\min}^A$ and

$\tilde{w}_{\sigma_2} = w^A_{\min}$. Then $p \cdot \tilde{w} \le p \cdot w$, so $\tilde{w}$ satisfies the budget constraint, and $U(\tilde{w}) > U(w)$ since $\tilde{w}^A_{\max} = \tilde{w}_{\sigma_1} > w^A_{\max}$ and $\tilde{w}^A_{\min} = w^A_{\min}$. This is a contradiction to optimality of w. Lemma D.2 implies that $\overline{\sigma} \in I$. The last statement of the lemma follows by observing that all these portfolios share the same price and utility. □

Lemma D.4. *Assume Lemma D.1 and let $\alpha < 1$. Then w is unambiguous, i.e. $w_\sigma = w_\nu$ for all $\sigma, \nu \in A$, if and only if (4.6) holds. In this case w is the only optimal portfolio. Condition (4.6) can only be satisfied if $\alpha \ge \frac{l-1}{l}$.*

Proof. Suppose $\overline{A} = \{\sigma\}$ and thus $\underline{A} = A \setminus \{\sigma\}$. Then, the first order conditions imply

$$\text{(D.2)} \qquad \frac{p_R}{\pi_R u'(w_R)} = \frac{p_\sigma}{(1-\alpha)(1-\sum_{R\in S\setminus A}\pi_R)u'(w^A_{\max})} = \frac{\sum_{\nu\in A\setminus\{\sigma\}} p_\nu}{\alpha(1-\sum_{R\in S\setminus A}\pi_R)u'(w^A_{\min})}$$

where R denotes any risky state among the m ones. Thus,

$$\text{(D.3)} \qquad \frac{p_\sigma}{\sum_{\nu\in A\setminus\{\sigma\}} p_\nu} = \frac{(1-\alpha)u'(w^A_{\max})}{\alpha u'(w^A_{\min})} < \frac{1-\alpha}{\alpha}$$

as $w^A_{\max} > w^A_{\min}$. Consequently, if there are no $\sigma \in A$ for which (D.3) is satisfied, i.e. if the condition (4.6) holds true, then w must be unambiguous. In order to prove necessity of (4.6), assume that (D.3) holds for some $\sigma \in A$. In the following we show that in this case the unambiguous portfolio cannot be optimal. To this end, suppose by contradiction that the unambiguous portfolio w is optimal and let $z := w^A_{\max} = w^A_{\min}$. Then $\epsilon = 0$ needs to maximize the function

$$F : \mathbb{R} \ni \epsilon \mapsto \alpha u(z-\epsilon) + (1-\alpha)u\left(z + \delta(\epsilon)\right)$$

over all $\epsilon \ge 0$, where $\delta(\epsilon) := \epsilon \frac{\sum_{\sigma\in A\setminus\{\sigma\}} p_\nu}{p_\sigma}$ is chosen such that the portfolio which invests $z - \epsilon$ in the states $\nu \in \underline{A}$, and $z + \delta(\epsilon)$ in the state σ satisfies the budget constraint (while the investment in the risky states is unaltered). F is a concave function and the first order condition reads

$$\frac{u'(z+\delta(\epsilon))}{u'(z-\epsilon)} = \frac{\alpha}{(1-\alpha)} \frac{p_\sigma}{\sum_{\sigma\in A\setminus\{\sigma\}} p_\nu}.$$

By assumption, the right hand side of the above equation is strictly smaller than 1. Hence, F attains its optimum for $\epsilon > 0$, which contradicts the optimality at 0 over all $\epsilon \ge 0$.

Finally, note that summing up (4.6) over all $\sigma \in A$ yields:

$$\alpha \sum_{\sigma\in A} p_\sigma \ge (1-\alpha)(l-1)\sum_{\nu\in A} p_\nu \quad \Leftrightarrow \quad \alpha \ge \frac{l-1}{l}.$$

□

Lemma D.5. *Assume Lemma D.1. If $\alpha = 1$, then w is unambiguous. If $\alpha = 0$, then there is no optimal portfolio.*

Proof. If $\alpha = 1$, then (4.2) is a maxmin agent and also $\tilde{\pi} \in \mathcal{C}_{\max}$. Hence, Lemma 4.7 proves the claim.

The optimization problem of a 0-MEU agent with the maximal set of priors $\mathcal{C}_{\max}$ is

$$\text{(D.4)} \qquad \sum_{R\in S\setminus A} \pi_R u(w_R) + (1 - \sum_{R\in S\setminus A} \pi_R) u(w^A_{\max}) \to \max$$
$$\text{subject to} \quad p\cdot w \le p\cdot e$$

where e denotes her initial endowment. Since the agent may go arbitrarily long in the ambiguous state σ with $w_\sigma = w^A_{\max}$ and satisfy the budget constraint by going arbitrarily short in an other ambiguous state, the optimal value in (D.4) cannot be attained. □

E Proof of Lemma 4.7

Let w be an optimal portfolio of the maxmin agent and assume that $w_\sigma \neq w_\eta$ for $\sigma, \eta \in A$. Consider the portfolio $\hat{w}$ given by $\hat{w}_R = w_R$ for any risky state $R \in S \setminus A$ and $\hat{w}_\sigma = z$ for any ambiguous state $\sigma \in A$ where

$$z := \frac{\sum_{\sigma\in A} p_\sigma\, w_\sigma}{\sum_{\sigma\in A} p_\sigma} = \frac{1}{l}\sum_{\sigma\in A} w_\sigma.$$

The portfolio $\hat{w}$ satisfies the budget constraint and

$$\begin{aligned} U(\hat{w}) &= \sum_{R\in S\setminus A} \pi_R\, u(w_R) + (1 - \sum_{R\in S\setminus A} \pi_R)\, u(z) \\ &> \sum_{R\in S\setminus A} \pi_R\, u(w_R) + \frac{1}{l}(1 - \sum_{R\in S\setminus A} \pi_R) \sum_{\sigma\in A} u(w_\sigma) \quad \geq \quad U(w) \end{aligned}$$

where the strict inequality follows from the strict concavity of u and the last inequality is due to $\tilde{\pi} \in \mathcal{C}$. This contradicts the optimality of w.

F Optimization in the partially concave case

Consider the optimization problem

$$\text{(F.1)} \qquad \max_{x\in C} U(x) \quad \text{subject to} \quad px \le pe$$

where $C \neq \emptyset$ is a convex subset of $\mathbb{R}^n$, $p, e \in \mathbb{R}^n$, and $U : \mathbb{R}^n \to \mathbb{R} \cup \{-\infty\}$ is a concave function with $\operatorname{dom} U = C$.

Lemma F.1. *If the optimal value in* (F.1) *is not* $+\infty$ *and if there exists at least one* $\bar{x} \in \operatorname{ri} C$ *with* $p\bar{x} \le pe$*, then there is a multiplier* $\lambda \ge 0$ *such that the supremum of* $h_\lambda(x) = U(x) - \lambda p(x - e)$*,* $x \in \mathbb{R}^n$*, is finite and equal to the optimal value in* (F.1)*. Moreover, suppose that* $\lambda > 0$ *and that* D *is the set of points* $x \in \mathbb{R}^n$ *where* h *attains its maximum intersected with the set of points satisfying* $px = pe$*, then* D *is the set of all optimal solutions to* (F.1)*.*

Proof. see Theorem 28.1 and Corollary 28.2.2 in Rockafellar (1997). □

Now suppose that agent n with choice criterium $U^n : \mathbb{R}^{|S|} \to \mathbb{R}$ maximizes her utility over all portfolios $w \in \mathbb{R}^{|S|}$ satisfying the budget constraint $pw \le pe^n$ for some $p \in \mathbb{R}^{|S|}$ with $p_i > 0$ for all $i = 1, \ldots, |S|$. Furthermore, assume that an optimal portfolio $\hat{w}$ exists and that $\hat{w} \in C$ for a convex set $C \subset \mathbb{R}^{|S|}$ such that the restriction U^n_C of U^n to C is concave. Then, we may view U^n_C as defined on all $\mathbb{R}^{|S|}$ by defining $U_C^n(x) := -\infty$ for $x \notin C$, and we are thus in the setting of Lemma F.1 where $\hat{w}$ is a solution to problem (F.1) with $U = U_C^n$. Hence, if there exists $x \in \operatorname{ri} C$ with $px \leq pe^n$, which is satisfied if for instance $\hat{w} \in \operatorname{ri} C$, then there exists a multiplier $\lambda \geq 0$ such that

$$U_C^n(\hat{w}) = \sup_{x \in \mathbb{R}^n} h_\lambda(x) \tag{F.2}$$

with h_λ as in Lemma F.1. If $C = C + \mathbb{R}_+ \cdot (1, 0, \ldots, 0)$ and given that the utility function u is strictly increasing we deduce that $\lambda > 0$, since otherwise

$$h_\lambda(\hat{w} + (1, 0, \ldots, 0)) = U_C^n(\hat{w} + (1, 0, \ldots, 0)) > U_C^n(\hat{w}).$$

Moreover, any solution $\hat{x}$ to the right hand side of (F.2) with $p\hat{x} = pe^n$ is a solution to the portfolio optimization problem, and in particular $\hat{w}$ is such a solution. Additionally, for any solution $\hat{x}$ to the right hand side of (F.2) we have for all $y \in \mathbb{R}^{|S|}$ that

$$U_C^n(y) - \lambda p(y - e^n) \leq U_C^n(\hat{x}) - \lambda p(\hat{x} - e^n)$$

which shows that

$$\lambda p \in \partial U_C^n(\hat{x}) \tag{F.3}$$

where $\partial U_C^n(w)$ denotes the supergradient of U_C^n at w, i.e.

$$\partial U_C^n(w) := \{\nu \in \mathbb{R}^{|S|} \mid \forall y \in \mathbb{R}^{|S|},\ U_C^n(y) \leq U_C^n(w) + \nu \cdot (y - w)\}.$$

References

Ahn, D., Choi, S., Gale, D., Kariv, S., 2014. Estimating ambiguity aversion in a portfolio choice experiment. Quantitative Economics 5 (2), 195–223.

Arrow, K., Hurwicz, L., 1972. An optimality criterion for decision making under ignorance. Basil Blackwell: Oxford, in C.F. Carter and J.L. Ford, Uncertainty and Expectations in Economics.

Bossaerts, P., Ghirardato, P., Guarnaschelli, S., Zame, W., 2010. Ambiguity in asset markets: theory and experiments. Review of Financial Studies 23, 1325–1359.

Boyle, P., Garlappi, L., Uppal, R., Wang, T., 2012. Keynes meets markowitz: The trade-off between familiarity and diversification. Management Science 58, 253–272.

Cao, H. H., Han, B., Hirshleifer, D., Zhang, H. H., 2011. Fear of the unknown: Familiarity and economic decisions. Review of Finance 15, 173–206.

Cao, H. H., Wang, T., Zhang, H., 2005. Model uncertainty, limited market participation, and asset prices. Review of Financial Studies 18, 1219–1251.

Chen, Y., Katuščák, P., Ozdenoren, E., 2007. Sealed bid auctions with ambiguity: Theory and experiments. Journal of Economic Theory 136, 513–535.

Chen, Z., Epstein, L., 2002. Ambiguity, risk, and asset returns in continuous time. Econometrica 70, 1403–1443.

Cubitt, R., van de Kuilen, G., Mukerji, S., 2014. Discriminating between models of ambiguity attitude: A qualitative test, mimeo, University of Oxford, United Kingdom.

De Castro, L., Pesce, M., Yannelis, N. C., 2017. Rational expectations equilibrium under ambiguity, working paper.

De Castro, L., Yannelis, N. C., 2016. Uncertainty, efficiency and incentive compatibility, working paper.

Dimmock, S. G., Kouwenberg, R., Wakker, P. P., 2016. Ambiguity attitudes in a large representative sample. Management Science 62, 1363–1380.

Dow, J., Werlang, S. R. d. C., 1992. Uncertainty aversion, risk aversion, and the optimal choice of portfolio. Econometrica 60, 197–204.

Easley, D., O'Hara, M., 2009. Ambiguity and nonparticipation: The role of regulation. Review of Financial Studies 22 (5), 1817–1843.

Eichberger, J., Grant, S., Kelsey, D., Koshevoy, G. A., 2011. The α-MEU model: A comment. Journal of Economic Theory 146, 1684–1698.

Ellsberg, D., 1961. Risk, ambiguity, and the savage axioms. Quarterly Journal of Economics 75, 643–669.

Epstein, L., Schneider, M., 2008. Ambiguity, information quality, and asset pricing. Journal of Finance 63, 197–228.

Epstein, L., Schneider, M., 2010. Ambiguity and asset markets. Annual Review of Financial Economics 2, 315–346.

Epstein, L. G., Miao, J., 2003. A two-person dynamic equilibrium under ambiguity. Journal of Economic Dynamics and Control 27, 1253 – 1288.

Ghirardato, P., Klibanoff, P., Marinacci, M., 1998. Additivity with multiple priors. Journal of Mathematical Economics 30, 405–420.

Ghirardato, P., Maccheroni, F., Marinacci, M., 2004. Differentiating ambiguity and ambiguity attitude. Journal of Economic Theory 118, 133–173.

Gilboa, I., Marinacci, M., 2013. Ambiguity and the bayesian paradigm. In: D. Acemoglu, M. A., Dekel, E. (Eds.), Advances in Economics and Econometrics: Theory and Applications, Tenth World Congress of the Econometric Society. New York: Cambridge University Press.

Gilboa, I., Schmeidler, D., 1989. Maxmin expected utility with non-unique prior. Journal of Mathematical Economics 18, 141–153.

He, W., Yannelis, N. C., 2015. Equilibrium theory under ambiguity. Journal of Mathematical Economics 61, 86 – 95.

Hurwicz, L., 1951a. Optimality criteria for decision making under ignorance, discussion Paper 370, Cowles Commission.

Hurwicz, L., 1951b. Some specification problems and application to econometric models. Econometrica 19, 343–344.

Illeditsch, P., 2011. Ambiguous information, portfolio inertia, and excess volatility. Journal of Finance 66, 2213–2247.

Jaffray, J.-Y., 1988. Application of linear utility theory to belief functions. Springer, in Uncertainty and intelligent systems: 1-8. Berlin.

Klibanoff, P., Mukerji, S., Seo, K., 2014. Perceived ambiguity and relevant measures. Econometrica 82, 1945–1978.

Knight, F., 1921. Risk, uncertainty and profit. Boston, MAss: Houghton, Mifflin.

Machina, M. J., Siniscalchi, M., 2014. Ambiguity and ambiguity aversion. In: Machina, M. J., Viscusi, W. K. (Eds.), Handbook of the Economics of Uncertainty. Elsevier.

Marinacci, M., 2002. Probabilistic sophistication and multiple priors. Econometrica 70, 755–764.

Olszewski, W., 2007. Preferences over sets of lotteries. Review of Economic Studies 74, 567–595.

Rockafellar, R., 1997. Convex analysis. Princeton landmarks in mathematics.

Siniscalchi, M., 2006. A behavioral characterization of plausible priors. Journal of Economic Theory 128, 91–135.

Trojani, F., Vanini, P., 2004. Robustness and ambiguity aversion in general equilibrium. Review of Finance 8, 279–324.

Uppal, R., Wang, T., 2003. Model misspecification and underdiversification. Journal of Finance 58, 2465–2486.