On capped product designs within variable annuities

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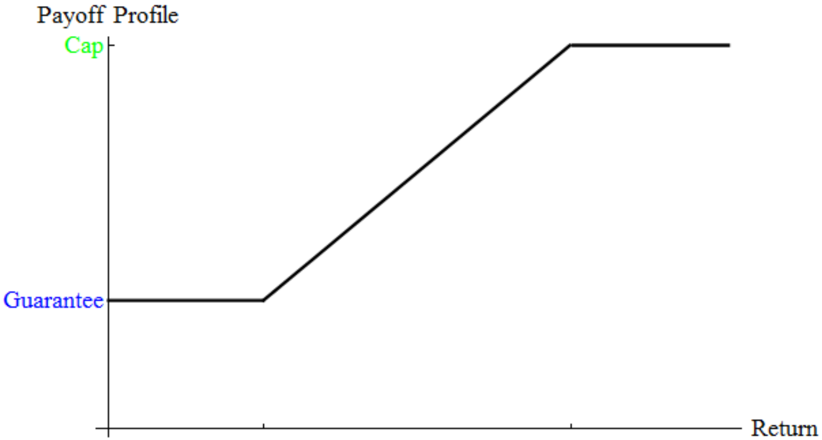
Variable annuities: unit-linked life insurance contracts

- Basic ingredient: buyer participates in the evolution of an investment in mutual funds
- In addition, the buyer can choose à la carte
  - Which protection feature he wants: the guarantee
  - Which protection style he wants: the guarantee types
  - Whether he wants to buy additional options (riders): fund switching rights, surrender rights, locally lock-in rights

► Recently, contracts with an additional cap on the investments at maturity of the contract became popular

Payoff variable annuity with guarantee and cap



Complex interplay between options

The perspective of the insurance company

- The buyer's choices influence the implicit charges
  - ▶ Thus, the pricing and risk management of the insurance company is affected

The perspective of the insured

- Additional riders/options influence the utility gained from the contract
  - ▶ But, the insured is also affected by the insurance companies pricing of the options

Research Question:

Why do we observe capped product designs?

- facilitates the risk management?
- brings benefits to the insured?

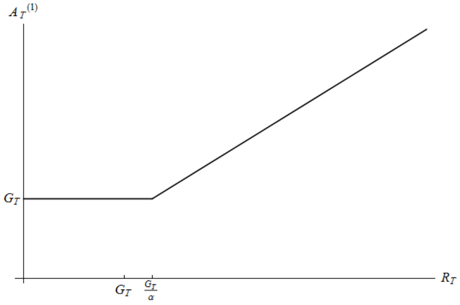
This talk: Mahayni and Schneider (2012b)

- Comparison of capped and uncapped product designs taking account of the rider to switch investment decisions (fund switching right)
- *Equivalently:* comparison of different manners of charging for the investment guarantee
- Implications of the contract design on the pricing of the insurance company and the utility of the insured

Payoff Profiles Simple Guarantee Contract

$$A_T^1 = \max\{Pe^{gT}, \alpha P R_T\}$$

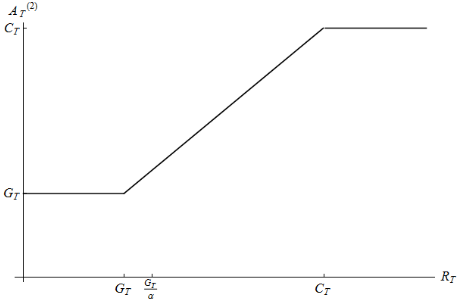
- R_T : return on the investment at maturity
- P : premium paid by the insured (normalized to one)
- $G_T = Pe^{gT}$: exogenous guarantee
- α investment fraction



Payoff Profiles Capped Contract

$$A_T^2 = \min\left\{\max\left\{Pe^{gT}, PR_T\right\}, Pe^{cT}\right\}$$

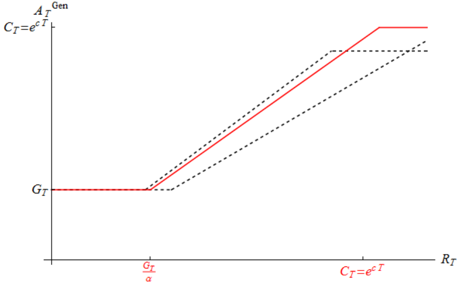
- R_T : return on the investment at maturity
- P : premium paid by the insured (normalized to one)
- $G_T = Pe^{gT}$: exogenous guarantee
- α investment fraction
- $C_T = Pe^{cT}$: cap



Payoff Profiles General Contract

$$A_T^{\text{Gen}} = \min\left\{\max\left\{Pe^{gT}, \alpha P R_T\right\}, Pe^{cT}\right\}$$

- R_T : return on the investment at maturity
- P : premium paid by the insured (normalized to one)
- $G_T = Pe^{gT}$: exogenous guarantee
- α investment fraction
- $C_T = Pe^{cT}$: cap



Without rider: Symmetric Information

- Commitment: Buyer of VA commits himself a priori to an investment strategy
- Commitment strategy is a constant mix in two risky and one risk-free asset (borrowing and short-selling constrained)

$$\frac{dV}{V_t} = \mu dt + \sigma_{\text{Inv}} dW_t$$

▶ Insurance company prices contracts fairly, i.e., present value of benefits must coincide with present value of contributions such that guarantee costs are calculated fairly

Without rider: Symmetric Information

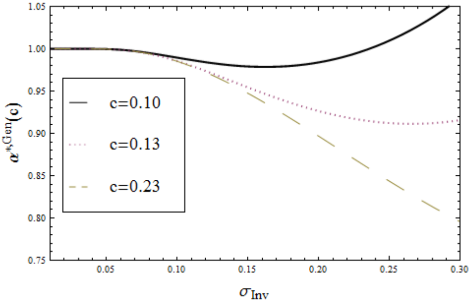
A **fair contract** satisfies

$$E_{P^*}\left[e^{-rT} A_T^{\text{Gen}}(\sigma_{\text{Inv}}, g, \alpha, c)\right] = P = 1$$

- σ_{Inv} : the portfolio volatility of the investment the insured commits himself to at contract initiation
- g : exogenously given guarantee rate
- α : investment fraction
- c : cap rate

Investment fraction without rider

Fair investment fraction



Parameter values: $r = 0.039$; $\sigma_1 = 0.30$; $\sigma_2 = 0.15$; $g = 0.00$; $T = 10$; $\rho = \pm 0.26$;

$$\sigma_{Inv} = \sqrt{\pi_1^2 \sigma_1^2 + \pi_2^2 \sigma_2^2 + 2\pi_1 \pi_2 \sigma_1 \sigma_2 \rho}$$

With rider: Asymmetric Information

- Insurer faces additional risk through uncertainty about insured's risk tolerance and future decisions
- Decisions typically depend on non-contractible information
- ▶ Insurance company prices contracts to be on the safe side
- We interpret the *information asymmetry* along the lines of an uncertain volatility model
- The provider knows that $\sigma_{t,\text{Inv}} \in [\sigma_{\text{min}}, \sigma_{\text{max}}]$ for all $t \in [0, T]$

$$dV_t^{\text{UVM}} = V_t^{\text{UVM}} \left(\mu_t^{\text{UVM}} dt + \sigma_t^{\text{UVM}} dW_t \right) \text{ where } V_0^{\text{UVM}} = \alpha^{Switch}.$$

With rider: Asymmetric information

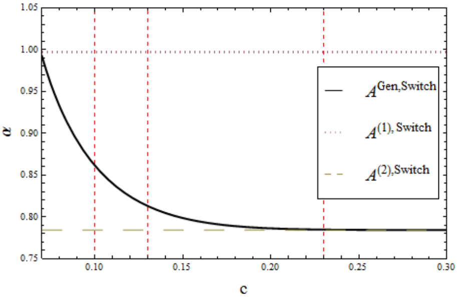
- The arbitrage-free price of A_T is not defined uniquely
- We consider the superhedging strategy which allows the provider to be on the safe side

$$A_t^{\text{Gen, Switch}} = v(t, V_t; A^{\text{Gen}}).$$

- v denotes the lowest upper price bound at time t
- v is the solution of a Black-Scholes-Barenblatt equation
- Generally, v cannot be obtained in closed-form
  - For the simple guarantee contract the above reduces to a Black-Scholes put price on σ_{max}
  - For the cap we rely on the results of Vanden (2006)

Investment fraction with rider

Investment fraction with rider

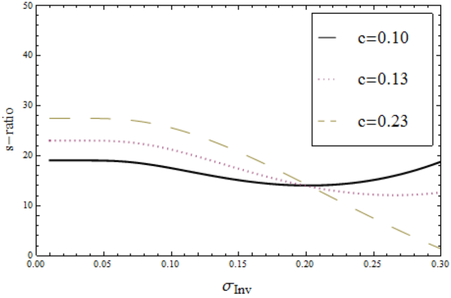


Parameter values: $r = 0.039$; $\sigma_1 = 0.30$; $\sigma_2 = 0.15$; $g = 0.00$; $T = 10$; $\rho = \pm 0.26$;

$$\sigma_{Inv} = \sqrt{\pi_1^2\sigma_1^2 + \pi_2^2\sigma_2^2 + 2\pi_1\pi_2\sigma_1\sigma_2\rho}$$

Sunk cost ratio

$$\frac{\alpha^* - \alpha^{\text{Switch}}}{\alpha^{\text{Switch}}}$$



Parameter values: $r = 0.039$; $\sigma_1 = 0.30$; $\sigma_2 = 0.15$; $g = 0.00$; $T = 10$; $\rho = \pm 0.26$;

$$\sigma_{Inv} = \sqrt{\pi_1^2 \sigma_1^2 + \pi_2^2 \sigma_2^2 + 2\pi_1 \pi_2 \sigma_1 \sigma_2 \rho}$$

Review: Mahayni and Schneider (2012a)

Findings: Simple Guarantee Contract

- The rider to switch gives an incentive to invest more aggressively (higher volatility than without)
- True benefits of the rider to switch are revealed in the presence of background risk
- Benefits of flexibility offset losses in investment premium

Optimization problem of the CRRA-insured: c fixed

Symmetric information

$$\max_{\pi \in \Pi} E\left[u(A_T^{\text{Gen}}(\alpha, c))\right] \text{ s.t. } \alpha = \alpha^{*,\text{Gen}}(c, \sigma_{\text{Inv}})$$

$$\sigma_{\text{Inv}} = \sqrt{\pi_1^2\sigma_1^2 + \pi_2^2\sigma_2^2 + 2\rho\pi_1\pi_2\sigma_1\sigma_2}.$$

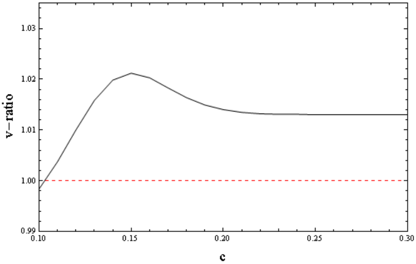
Asymmetric information

$$\max_{\pi \in \Pi} E\left[u(A_T^{\text{Gen}}(\alpha, c))\right] \text{ s.t. } \alpha = \alpha^{*,\text{Gen,Switch}}(c).$$

Expected utility can be represented in closed-form but optimization needs numerical method

Distortions on the volatility

$$v - ratio = \frac{\sigma^{*,\text{Switch}}(c)}{\sigma^{*}(c)}$$

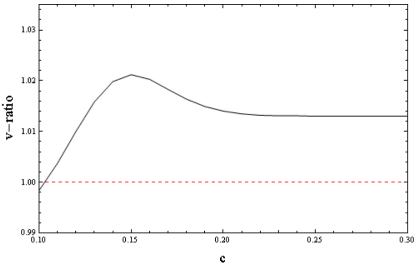


Parameter values: $r = 0.039$; $\sigma_1 = 0.29$; $\mu_1 = 0.08$; $\sigma_2 = 0.15$; $\mu_2 = 0.10$; $g = 0.00$; $T = 10$; $\rho = -0.26$;

$$\sigma_{\text{Inv}} = \sqrt{\pi_1^2\sigma_1^2 + \pi_2^2\sigma_2^2 + 2\rho\pi_1\pi_2\sigma_1\sigma_2}, \gamma = 2$$

Distortions on the volatility

$$v-ratio = \frac{\sigma^{*,\text{Switch}}(c)}{\sigma^{*}(c)}$$



Certainty Equivalent:

$$CE_T = \left((1-\gamma)E_P\left[u(A_T^{\text{Gen}})\right]\right)^{\frac{1}{1-\gamma}}$$

	$A^{*,\text{Gen}}(c)$	$A^{*,\text{Gen,Switch}}(c)$
	g=0.00	g=0.00
c=10%		
μ_{Inv} p.a.	8.66%	8.62%
σ_{Inv} p.a.	11.0%	10.9%
CE	2.00	1.85
c=19%		
μ_{Inv} p.a.	8.66%	8.69%
σ_{Inv} p.a.	11.0%	11.2%
CE	2.07	1.66
c=∞		
μ_{Inv} p.a.	8.66%	8.69%
σ_{Inv} p.a.	11.0%	11.2%
CE	2.07	1.65

Optimization problem of the CRRA-insured: c endogenous

Symmetric information

- Optimizing over c yields that the optimal contract is a simple guarantee contract ▶ El Karoui et al. (2005)
- The optimal commitment strategy is the Merton strategy, i.e., a constant mix strategy
- Additional cap leads to loss in utility

Asymmetric information

- No closed-form solution possible
- Numerically tractable relying on closed-form representation of expected utility

Theoretical predictions

- Symmetric information

$$\max_c \max_{\pi\in\Pi} E\left[u(A_T^{\text{Gen}}(\alpha, c))\right] = \max_{\pi\in\Pi} E\left[u(A_T^{\text{Gen}}(\alpha, \infty))\right]$$
$$= \max_{\pi\in\Pi} E\left[u(A_T^{(1)}(\alpha))\right].$$

- Asymmetric information

$$\max_c \max_{\pi\in\Pi} E\left[u(A_T^{\text{Gen,Switch}}(\alpha, c))\right] > \max_{\pi\in\Pi} E\left[u(A_T^{\text{Gen,Switch}}(\alpha, \infty))\right]$$
$$= \max_{\pi\in\Pi} E_P\left[u(A_T^{(1),\text{Switch}}(\alpha))\right]$$

Utility loss caused by sunk costs and payoff distortions

Loss Rate

$$L_T^{\text{Switch}} = \frac{1}{T} \ln\left(\frac{CE^{*,\text{Gen,Switch}}}{CE^{*,\text{Gen}}}\right)$$

$$\Rightarrow \quad L_T^{\text{Switch}} = L_T^{\text{Sunk}} + L_T^{\text{Dis}}$$

Utility loss of sunk costs and payoff distortions

γ	c^*	$c^{*,\text{Switch}}$	σ^*_{Inv}	$\sigma^{*,\text{Switch}}_{\text{Inv}}$	v %	L_T^{Switch} bp	$\frac{L_T^{\text{Sunk}}}{L_T^{\text{Switch}}}$ %	$\frac{L_T^{\text{Dis}}}{L_T^{\text{Switch}}}$ %
				Panel $g = 0.00$				
2	∞	0.090	0.110	0.110	100.0	107.10	56.11	43.89
3	∞	0.081	0.109	0.109	100.0	92.50	48.10	51.90
4	∞	0.069	0.108	0.072	67.0	58.90	15.56	84.44
5	∞	0.069	0.087	0.068	78.0	38.30	12.77	87.23
6	∞	0.069	0.072	0.063	88.0	17.42	26.52	73.48

Discussion and Conclusion

- Capped contract has merits under asymmetric information due to the rider to switch
  - Offsets unwanted distortions of the investment decision
  - Without cap the investor conducts a riskier strategy due to the worst case pricing
  - Cap gives an opportunity to mitigate the sunk costs
- Implication beyond the rider to switch
  - Seemingly suboptimal change in contract design can be beneficial to offset utility losses due to other sources

▶ What about the risk management?