Economic Neutral Position: How to best replicate not fully replicable liabilities

Preprint: http://arxiv.org/abs/1704.08523
(-> joint work with Markus Popp)

Zürich – Hannover Workshop

November 2, 2017

Dr. Andreas Kunz

Different approaches for Internal Models

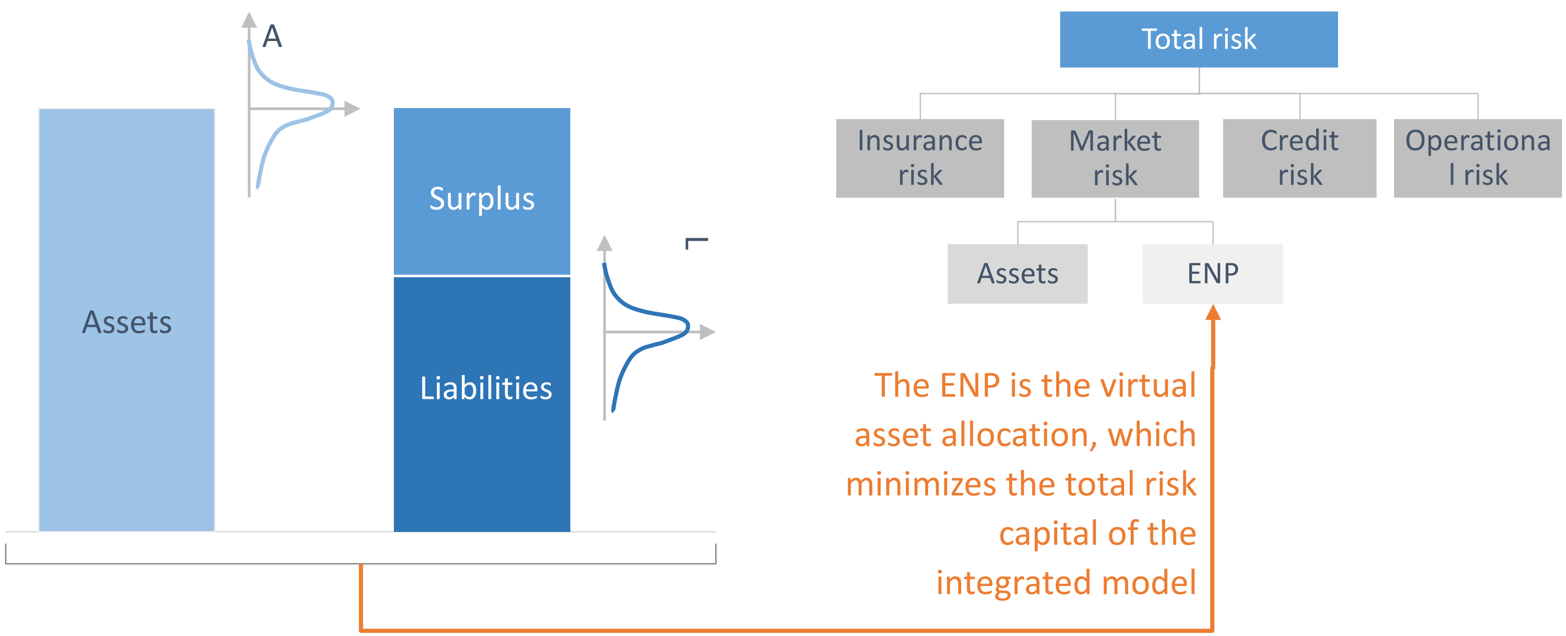
Integrated vs. Modular Risk Model

Integrated Risk Model

- Joint stochastics of all risk drivers (assets & liabilities)
- VaR from surplus distribution

Modular Risk Model (Industry Standard)

- Separate modules for each risk category
- Aggregation of risk modules yields Top Risk
- Introduction of Replicating Portfolios for market risk module



The choice of the Replicating Portfolio must ensure consistency across the different risk modules of the modular risk model

Possible Choices for Replicating Portfolios

Economic Neutral Position replicates also a certain fraction of the non-hedgeable SCR (on top of the technical provisions)

1 SII Standard Formula

Only BEL and Other Positions are subject to market risk shocks

2 Replicating Portfolio (RP)

Risk Margin is included based on:

a) only discount effect <u>or</u>

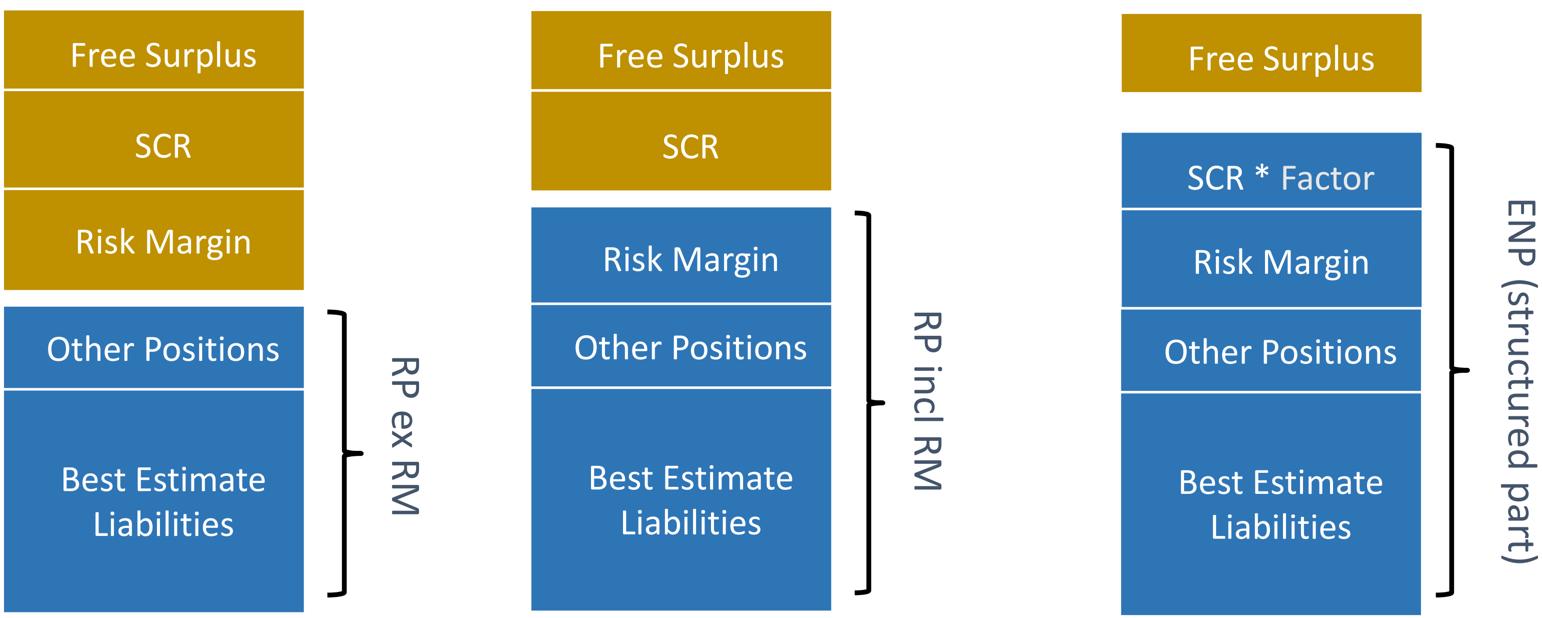
b) full interest rate sensitivity

3 Economic Neutral Position (ENP)

Surplus structure is included, based on

a) non-hedgeable insurance SCR <u>and</u>

b) **scaling factor for the risk-minimal solution**



Modular Model matches risk figures of Integrated Model — Market risk SCR

Protection of solvency ratio — Asset steering

Illustrative Example

What is the risk-minimal asset allocation?

Initial setup

- EUR company has USD liability of €100 and €150 assets in €-cash
- How much USD cash shall be bought in order to be risk-minimal?

$$S_0 = A_\$ + A_€ - L_0$$

After shock event

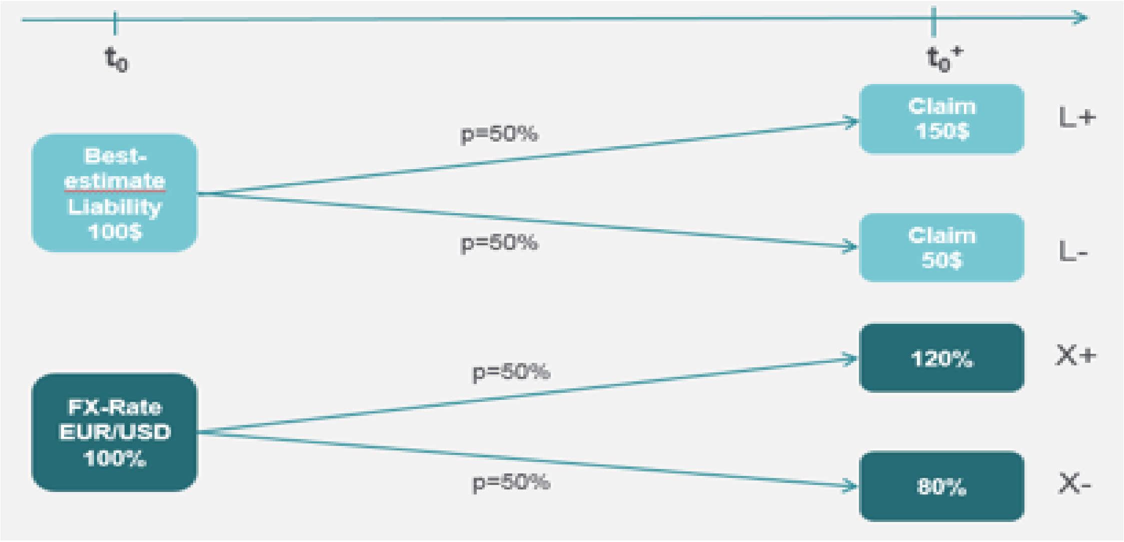
- Simultaneous shock event:
  - L = liability size
  - X = $/€ exchange rate

$$S_{0^+} = A_\$ \, X - L \, X + A_€$$

P&L effect

- Loss $Z = -(S_{0^+} - S_0)$
- Compute largest loss depending on the asset allocation

$$Z = (A_\$ - L_0)(1 - X) + (L - L_0)X$$



Scenario	Loss Z in EUR A=100	A=150
L+ X+	**60**	**50**
L+ X-	40	50
L- X-	-40	-30
L- X+	-60	-70

Investing the best estimate US$ exposure of the liabilities is not risk-minimal

Definition of the ENP

Introduction of the risk drivers for the general model

Assets (=ENP)

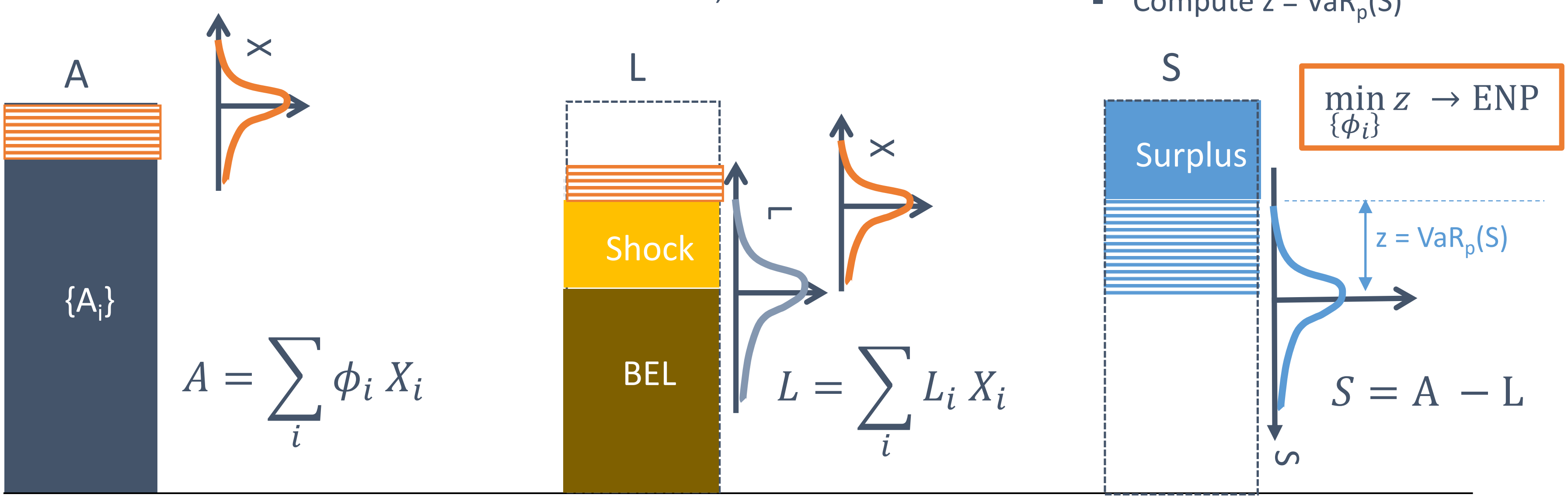
- Synthetic Zero Coupon Bonds for different maturities and currencies with market value A_i
- Subject to market risk drivers X_i incl. FX, interest rate & inflation risk

Liabilities

- Liabilities are subject to
  - insurance risk drivers: mortality, lapse, etc.
  - market risk drivers: FX, interest & inflation

Surplus

- Surplus = A - L
- Subject to both market and insurance risk
- Subject to asset allocation
- Compute $z = VaR_p(S)$



$$X_i \sim \frac{f_k}{f_k^0} \cdot e^{-(r_{t,k}-r_{t,k}^0)t} \cdot e^{(j_{t,k}-j_{t,k}^0)t}$$

- f_k is the exchange rate of currency k to €.
- $r_{t,k}$ is the nominal interest rate for maturity t and currency k
- $j_{t,k}$ is the stochastic implied inflation rate for t and k

Definition of the ENP

Assumptions for the general model

The Model

- Surplus after 1 year

$$S(\phi) = A_0 + \sum_i \phi_i \cdot (X_i - X_{i,0}) - X_i \cdot L_i$$

- Elimination of mean value by change of variables:

$$\boldsymbol{L} \to \boldsymbol{L} - \mathbb{E}[\boldsymbol{L}], \quad \boldsymbol{\phi} \to \boldsymbol{\phi} - \mathbb{E}[\boldsymbol{L}],$$

WLoG: $\mathbb{E}[X_i] = X_{i,0} = 1,\ A_0 = \mathbb{E}[L_i] = 0$

- Surplus rewritten (with zero mean)

$$S(\phi) = \sum_i \phi_i \cdot (X_i - 1) - X_i \cdot L_i$$

- Risk minimal asset allocation ϕ^*

$$\varrho[S(\phi^*)] = \min_{\phi} \varrho[S(\phi)],\ \rho \in \{VaR_\alpha, ES_\alpha\}$$

Assumptions

- Liability exhibits product structure $\sum_i X_i \cdot L_i$
- Non-hedgeable claim sizes L_i are **independent** from the tradeable assets X_i .
- The market risk factors X_i are **positive**

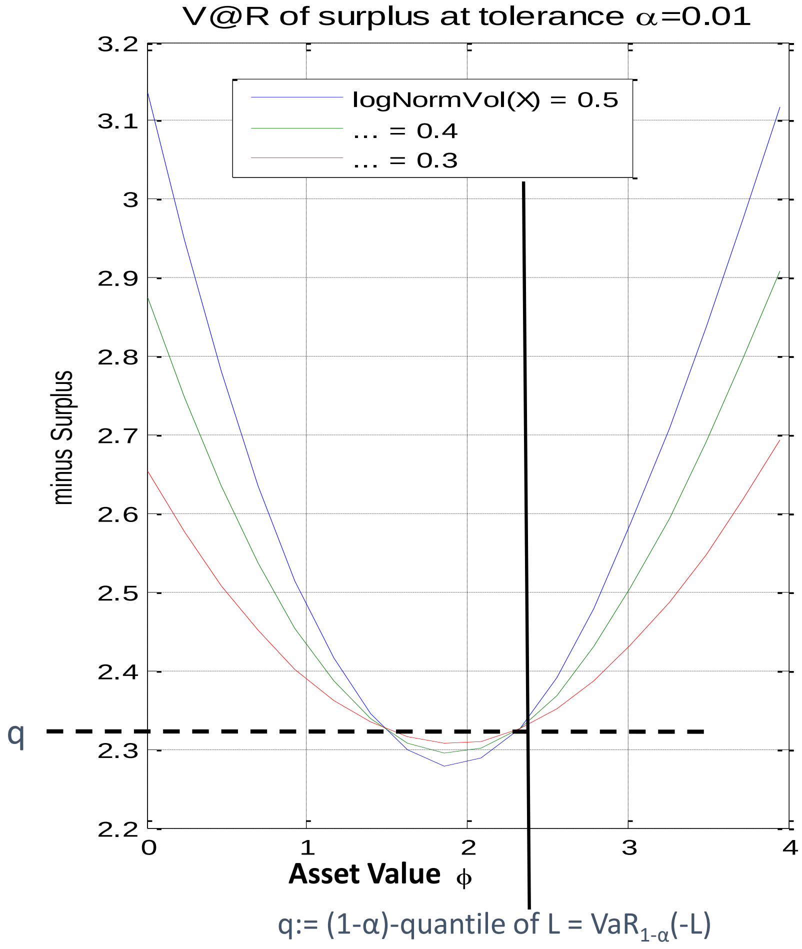
Examples

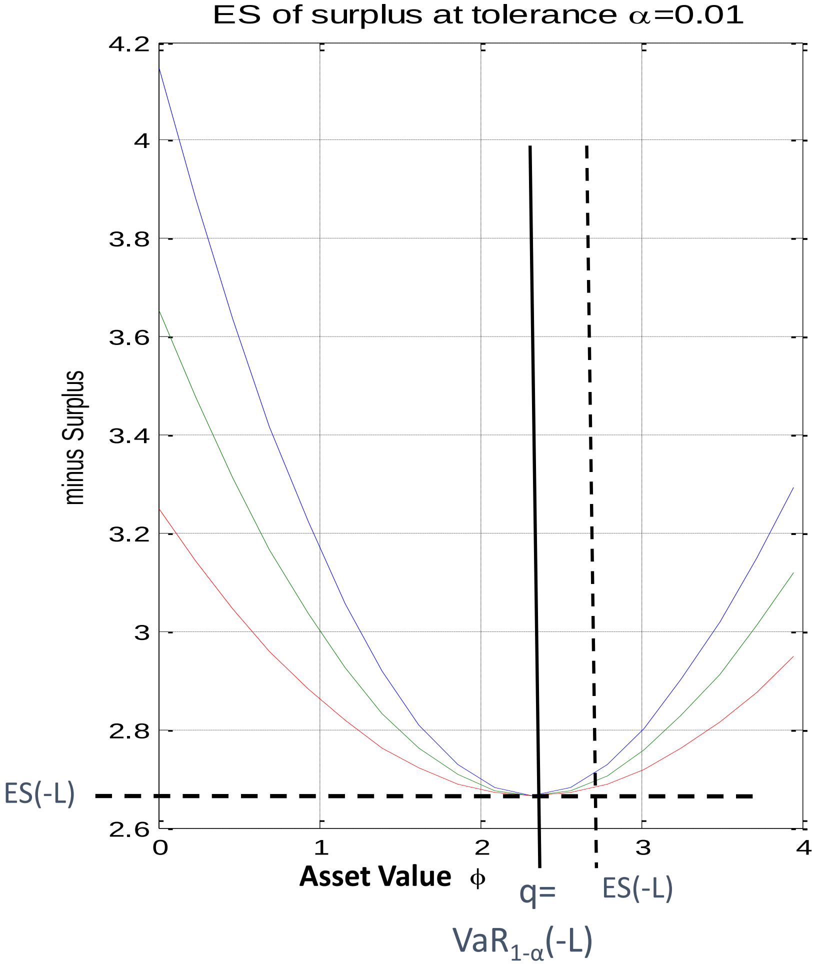
- Insurance Non-Life: L = US-NatCat exposure, X = EUR/USD FX-rate
- Insurance Life: L = survival benefit in 20 years, X = 20y discount rate
- CVA for non-collateralized derivative with CP for which no CDS exists: L = LGD * PD of CP, X = discounted PFE at year 1

The ENP is the asset allocation, which minimizes the total value-at-risk, i.e. ENP = ϕ^*

Simulation Study (one-dimensional case)

Value-at-Risk and Expected Shortfall*





*) $L \sim \mathcal{N}(0,1)$, $X \sim \mathcal{LN}(\mu, \sigma_x)$ with $\mu = -\frac{\sigma_x^2}{2}$, # simulations = *1e7*

Particular asset value in the one-dimensional case

ϕ equals value-at-risk of pure insurance risk component

Theorem [particular asset value] If $q := F_L^{-1}(1-\alpha) = \mathrm{VaR}_\alpha[-L]$ units are initially invested in X, i.e. $\phi = q$, then

a) $\rho[S(q)] = \rho[-L]$ for $\rho \in \{\mathrm{VaR}_\alpha, \mathrm{ES}_\alpha\}$.

b) $$\Big(\partial_\phi\, \rho[S(\phi)]\Big)_{|\phi=q} = \begin{cases} (-1)\cdot\Big(\mathbb{E}[X^{-1}]^{-1} - 1\Big) \geq 0 & \text{if } \rho = \mathrm{VaR}_\alpha\,, \\ 0 & \text{if } \rho = \mathrm{ES}_\alpha \end{cases}$$

and the inequality becomes strict if X is not constant.

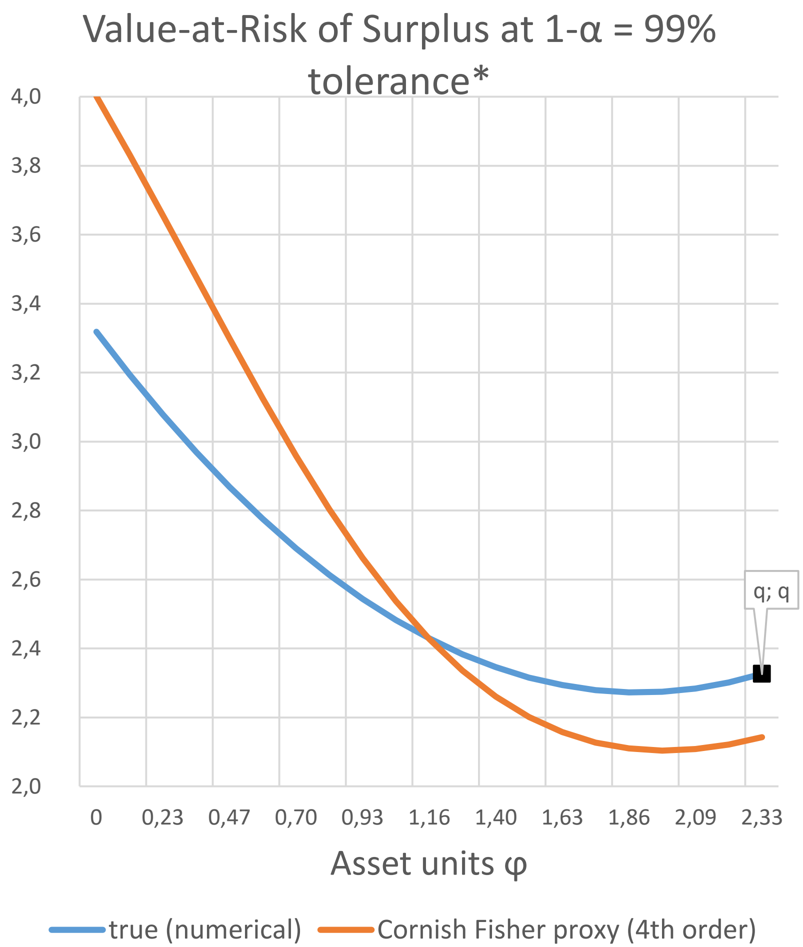
c) $\phi \mapsto \mathrm{ES}_\alpha[S(\phi)]$ is convex with global minimum $\mathrm{ES}_\alpha[-L]$ at $\phi^* = q$.

Sketch of Proof of a) for VaR: Key ingredient: positivity of X!

$$\begin{aligned} \{S(q) \leq -q\} &= \{q\cdot(X-1) - X\cdot L \leq -q\} \\ &= \{X\cdot(q-L) \leq 0\} = \{q - L \leq 0\} = \{L \geq q\}\,. \end{aligned}$$

Hence $\mathbb{P}\,(S(q) \leq -q) = 1 - F_L(q) = \alpha$, which implies $\mathrm{VaR}_\alpha\,[S(q)] = q$.

Classical quantile expansion techniques
Naive application of Cornish Fisher not adequate



- Cornish–Fisher (CF) expansion: approximates quantiles of probability distribution via its cumulants with normal distribution as base.
- CF expansion up to 4th order (orange line in graph)
- Observation: CP expansion does not match particular asset value $\phi = q$
- Reasons: due to the product structure of the liability skew and kurtosis of the surplus distribution differ considerably from those of the normal distribution

➢ Normal distribution is the wrong base distribution

* $L \sim \mathcal{N}(0,1)$, $X \sim \mathcal{LN}(\mu, \sigma_x)$ with $\sigma_x = 0.5$ and $\mu = -\frac{\sigma_x^2}{2}$

Expansion Results (multi-dimensional setting)

Preparation

General Expansion Result

- **Proposition**: Expansion of distribution $V + Y$:

$$\mathbb{P}(V + \mathrm{Y} \leq \mathrm{z}) = \sum_{r \geq 0} \frac{1}{r!} \cdot (-D_z)^r \, \mathbb{E}[Y^r \cdot \chi_{V \leq z}]$$

- Note: if X and Y independent, special case of Gram/Charlier series with V as base distribution

$$f_{V+Y}(z) = \sum_{r \geq 0} m_r(Y) \frac{(-D_z)^r}{r!} \cdot f_V(z)$$

- Proof: $\phi_{Y+V}(t) = \mathbb{E}\left[e^{it} \cdot \mathbb{E}[e^{iVt}|Y]\right]$, Taylor expansion e^{iYt}, plus invers Fourier trafo
- Intuition: $\chi_{v+y \leq z} = H(z - v - y)$ *"Heavyside"*

$$= H(z - v) - \delta(z - v) \cdot y + \frac{1}{2} \delta'^{(z-v)} \cdot y^2 + \cdots$$

$$= \chi_{v \leq z} - D_z \, \chi_{v \leq z} \cdot y + \frac{1}{2} D_z^2 \, \chi_{v \leq z} \cdot y^2 + \cdots$$

Application to ENP setting

- Rewrite Surplus $\mathrm{S}(\phi) = \mathrm{V} + \mathrm{Y}$ with

$$\mathrm{V} = -\sum_i L_i = -\langle \mathbf{1}, \boldsymbol{L} \rangle,$$

$$\mathrm{Y} = \langle \boldsymbol{X} - \mathbf{1}, \boldsymbol{\phi} - \boldsymbol{L} \rangle$$

- Apply **Prop:** $\alpha \doteq \mathbb{P}(\mathrm{S}(\phi) \leq -\mathrm{z}) = \bar{F}_{\langle \mathbf{1}, \boldsymbol{L} \rangle}(z)$ $+ \frac{1}{2} D_z^2 \, \mathbb{E}\left[\langle \boldsymbol{X} - \mathbf{1}, \boldsymbol{\phi} - \boldsymbol{L} \rangle^2 \cdot \chi_{\langle \mathbf{1}, \boldsymbol{L} \rangle \geq z}\right] + \ldots$ (*)
- Expand the quantile $z = z(\phi)$ $= z_0 + z_1 + z_2 \cdots, \; z_0 \sim \sigma^i$, where $\sigma = \max_i \sqrt{V[\ln X_i]}$ is the log-normal asset volatility
- Insert this expansion in (*) and solve for increasing orders in σ.

Expansion Results for Value at Risk
Up to second order (multi-variate setting)

Denote: $q := \mathrm{VaR}_\alpha[-\langle \mathbf{1}, \mathbf{L}\rangle] = F^{-1}_{\langle \mathbf{1},\mathbf{L}\rangle}(1-\alpha)$, $\boldsymbol{\Sigma}$ covar matrix of $\mathbf{X}$, $\mathbf{D} = \left(\frac{1}{\sqrt{n}}\mathbf{1}\middle|\mathbf{1}^\perp\right) \in SO(n)$, $g(\mathbf{m}) := f_{\mathbf{L}}(\mathbf{Dm})$ and

$$\begin{aligned} \mathbf{h}(z) &:= \frac{1}{\sqrt{n}}\int_{\mathbb{R}^{n-1}} \bar{\mathbf{m}} \cdot g\left(\tfrac{z}{\sqrt{n}}, \bar{\mathbf{m}}\right) d\bar{\mathbf{m}}, \\ h_{\mathbf{A}}(z) &:= \frac{1}{\sqrt{n}}\int_{\mathbb{R}^{n-1}} \langle \bar{\mathbf{m}}, \mathbf{A}\bar{\mathbf{m}}\rangle \cdot g\left(\tfrac{z}{\sqrt{n}}, \bar{\mathbf{m}}\right) d\bar{\mathbf{m}} \quad (\mathbf{A} \in \mathbb{R}^{n-1\times n-1}) . \end{aligned}$$

Theorem: Expansion of $\mathrm{VaR}_\alpha[S(\phi)]$ up to 2nd order in log-normal volatility σ of $\mathbf{X}$:

$$\begin{aligned} \mathrm{VaR}_\alpha[S(\phi)] &= q + \frac{1}{2f_{\langle \mathbf{1},\mathbf{L}\rangle}(q)} \cdot D_q^2\, \mathbb{E}_{\mathbf{L}}\Big[\langle \boldsymbol{\phi} - \mathbf{L}, \boldsymbol{\Sigma}, \boldsymbol{\phi} - \mathbf{L}\rangle \cdot \chi_{\langle \mathbf{1},\mathbf{L}\rangle > q}\Big] + o(\sigma^2) \\ &= q - \frac{1}{2f_{\langle \mathbf{1},\mathbf{L}\rangle}(q)} \cdot \Big\{ f'_{\langle \mathbf{1},\mathbf{L}\rangle}(q) \cdot \langle \boldsymbol{\phi} - \tfrac{q}{n}\cdot \mathbf{1}, \boldsymbol{\Sigma}, \boldsymbol{\phi} - \tfrac{q}{n}\cdot\mathbf{1}\rangle \\ &\quad -2\cdot\langle \mathbf{1}^\perp \mathbf{h}'(q), \boldsymbol{\Sigma}, \boldsymbol{\phi} - \tfrac{q}{\sqrt{n}}\cdot\mathbf{1}\rangle + \tfrac{2}{n}\langle \mathbf{1}^\perp \mathbf{h}(q), \boldsymbol{\Sigma}\cdot\mathbf{1}\rangle + h_{\mathbf{1}^{\perp\prime}\boldsymbol{\Sigma}\mathbf{1}^\perp}(q)\Big\} \\ &\quad -\tfrac{1}{n}\cdot\langle \mathbf{1}, \boldsymbol{\Sigma}, \boldsymbol{\phi} - \tfrac{q}{n}\cdot\mathbf{1}\rangle + o(\sigma^2) . \end{aligned}$$

If $f'_{\langle \mathbf{1},\mathbf{L}\rangle}(q) \neq 0$ and $\boldsymbol{\Sigma}$ is invertible, the risk minimal ϕ is

$$\boldsymbol{\phi}^* = \frac{1}{n}\cdot\left(q + \frac{f_{\langle \mathbf{1},\mathbf{L}\rangle}(q)}{f'_{\langle \mathbf{1},\mathbf{L}\rangle}(q)}\right)\cdot\mathbf{1} + \frac{1}{f'_{\langle \mathbf{1},\mathbf{L}\rangle}(q)}\cdot\mathbf{1}^\perp\cdot\mathbf{h}'(q) .$$

Expansion Results

Up to second order (multi-variate setting)

Corollary: a) Expansion of $\text{ES}_\alpha[S(\phi)]$ up to 2nd order in log-normal asset volatility σ:

$$\begin{aligned}
\text{ES}_\alpha[S(\phi)] &= \text{ES}_\alpha[-\langle \mathbf{1}, \mathbf{L}\rangle] - \tfrac{1}{2\alpha} \cdot D_q \, \mathbb{E}_\mathbf{L}\Big[\langle \boldsymbol{\phi} - \mathbf{L}, \boldsymbol{\Sigma}, \boldsymbol{\phi} - \mathbf{L}\rangle \cdot \chi_{\langle \mathbf{1}, \mathbf{L}\rangle > q}\Big] + o(\sigma^2) \\
&= \text{ES}_\alpha[-\langle \mathbf{1}, \mathbf{L}\rangle] + \tfrac{1}{2\alpha} \cdot \Big\{ f_{\langle \mathbf{1}, \mathbf{L}\rangle}(q) \cdot \langle \boldsymbol{\phi} - \tfrac{q}{n} \cdot \mathbf{1}, \boldsymbol{\Sigma}, \boldsymbol{\phi} - \tfrac{q}{n} \cdot \mathbf{1}\rangle \\
&\quad -2 \cdot \langle \mathbf{1}^\perp \cdot \mathbf{h}(q), \boldsymbol{\Sigma}, \boldsymbol{\phi} - \tfrac{q}{n} \cdot \mathbf{1}\rangle + h_{\mathbf{1}^{\perp\prime} \boldsymbol{\Sigma} \mathbf{1}^\perp}(q) \Big\} + o(\sigma^2)\,.
\end{aligned}$$

b) If $\boldsymbol{\Sigma}$ is invertible, the risk minimal $\boldsymbol{\phi}$ is

$$\boldsymbol{\phi}^* = \frac{q}{n} \cdot \mathbf{1} + \frac{1}{f_{\langle \mathbf{1}, \mathbf{L}\rangle}(q)} \cdot \mathbf{1}^\perp \cdot \mathbf{h}(q)\,.$$

Sketch of Poof: $\text{ES}_\alpha[S(\phi)] = \frac{1}{\alpha} \int_0^\alpha \text{VaR}_\beta[S(\phi)]\, d\beta$.

For any rv with density $f > 0$, $G \in \mathcal{C}^1(\mathbb{R}, \mathbb{R})$, $\alpha \in (0,1)$

$$\int_0^\alpha \frac{G'(q_\beta)}{f(q_\beta)}\, d\beta = -G(q_\beta)\,, \quad \text{where } q_\beta := F^{-1}(1-\beta)\,.$$

Expansion Results for Value at Risk
Total Optimal Asset Amount

Corrolary: Total optimal asset amount

$$\sum_i \phi_i^* = \langle \mathbf{1}, \boldsymbol{\phi}^* \rangle = q + \begin{cases} f_{\langle \mathbf{1},\mathbf{L}\rangle}(q)/f'_{\langle \mathbf{1},\mathbf{L}\rangle}(q) & \text{if} \quad \rho = \mathrm{VaR}_\alpha\,, \\ 0 & \text{if} \quad \rho = \mathrm{ES}_\alpha\,. \end{cases}$$

Further $\sum_i \phi_i^*$ coincides with the optimal asset value ϕ_0^* in the *associated single-asset case* where $X_i = X_1$.

Corrolary: If $\rho = \mathrm{VaR}_\alpha$ and $\mathbf{L} \sim \mathcal{N}$, then $\phi_0^*/q = 1 - u_\alpha^{-2}$, where $u_\alpha := F^{-1}_{\mathcal{N}(0,1)}$. In Solvency II ($1-\alpha = 99.5\%$) we obtain $\phi_0^*/q = 85\%$.

Theorem: Assume $\mathbf{L} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}^{\boldsymbol{L}})$. Then for $\rho \in \{\mathrm{VaR}_\alpha, \mathrm{ES}_\alpha\}$ the asset amounts ϕ_i^* minimizing $\rho[S(\boldsymbol{\phi})]$ expanded up to second order in log-normal asset volatility follow the *covariance allocation principles* with respect to $\mathbf{L}$, i.e.

$$\phi_i^* = \frac{\Sigma_{ii}^{\mathbf{L}} + \sum_{j \neq i} \Sigma_{ij}^{\mathbf{L}}}{\langle \mathbf{1}, \boldsymbol{\Sigma}^{\boldsymbol{L}} \mathbf{1} \rangle} \cdot \phi_0^* \qquad (i = 1, \ldots, n)\,,$$

where $\langle \mathbf{1}, \boldsymbol{\Sigma}^{\boldsymbol{L}} \mathbf{1} \rangle$ is the total variance of $\sum_i L_i$.

Expansion Results up to Third Order
Univariate setting

Theorem [1-dim case] Denoting by μ_3 the centered normalized moment of $\ln X$, the expansion of $\rho[S(\phi)]$ up to 3rd order in log-normal asset volatility σ read:

a) Value-at-risk case:

$$\mathrm{VaR}_\alpha[S(\phi)] = q - \frac{1}{f_L(q)} \cdot \left\{ \left((\phi - id)^2 f_L\right)'(q) \cdot \frac{\sigma^2}{2} + \left((\phi - id)^3 f_L'\right)'(q) \cdot \frac{\sigma^3 \mu_3}{6} \right\} + o(\sigma^3),$$

If $\mu_3 \cdot f_L''(q) \neq 0$, this expansion is (locally) minimized by
$\phi^* = q + \frac{1}{f_L''(q)} \left((1-\delta) f_L'(q) - \sqrt{(1-\delta)^2 f_L'(q)^2 + 2\delta f_L''(q) f_L(q)} \right) \; (\delta := \frac{1}{\sigma \mu_3})$.

b) Expected shortfall case:

$$\mathrm{ES}_\alpha[S(\phi)] = \mathrm{ES}_\alpha[-L] + \frac{\sigma^2}{2\alpha} \cdot (\phi - q)^2 \cdot f_L(q) + \frac{\sigma^3 \mu_3}{6\alpha} \cdot (\phi - q)^3 \cdot f_L'(q) + o(\sigma^3),$$

which is minimized by $\phi^* = q$.

Numerical analysis vs. theoretical findings

Risk of the surplus as a function of the asset units ϕ

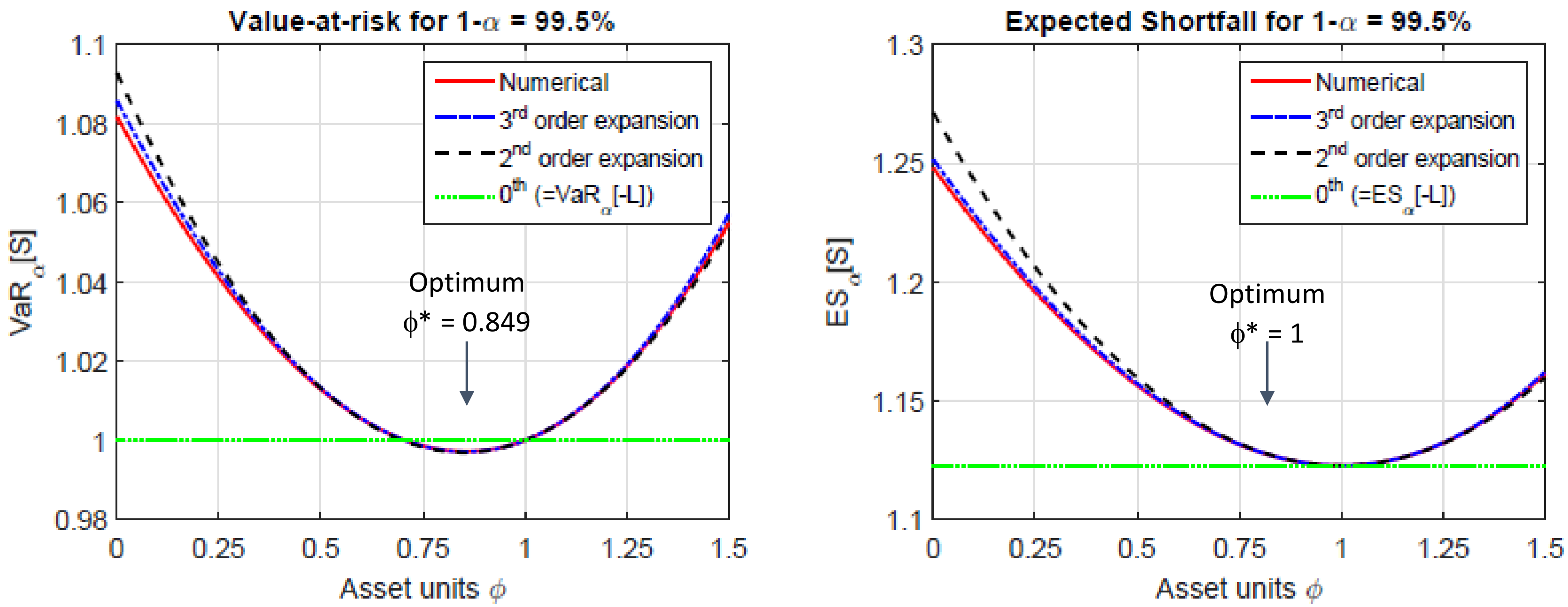
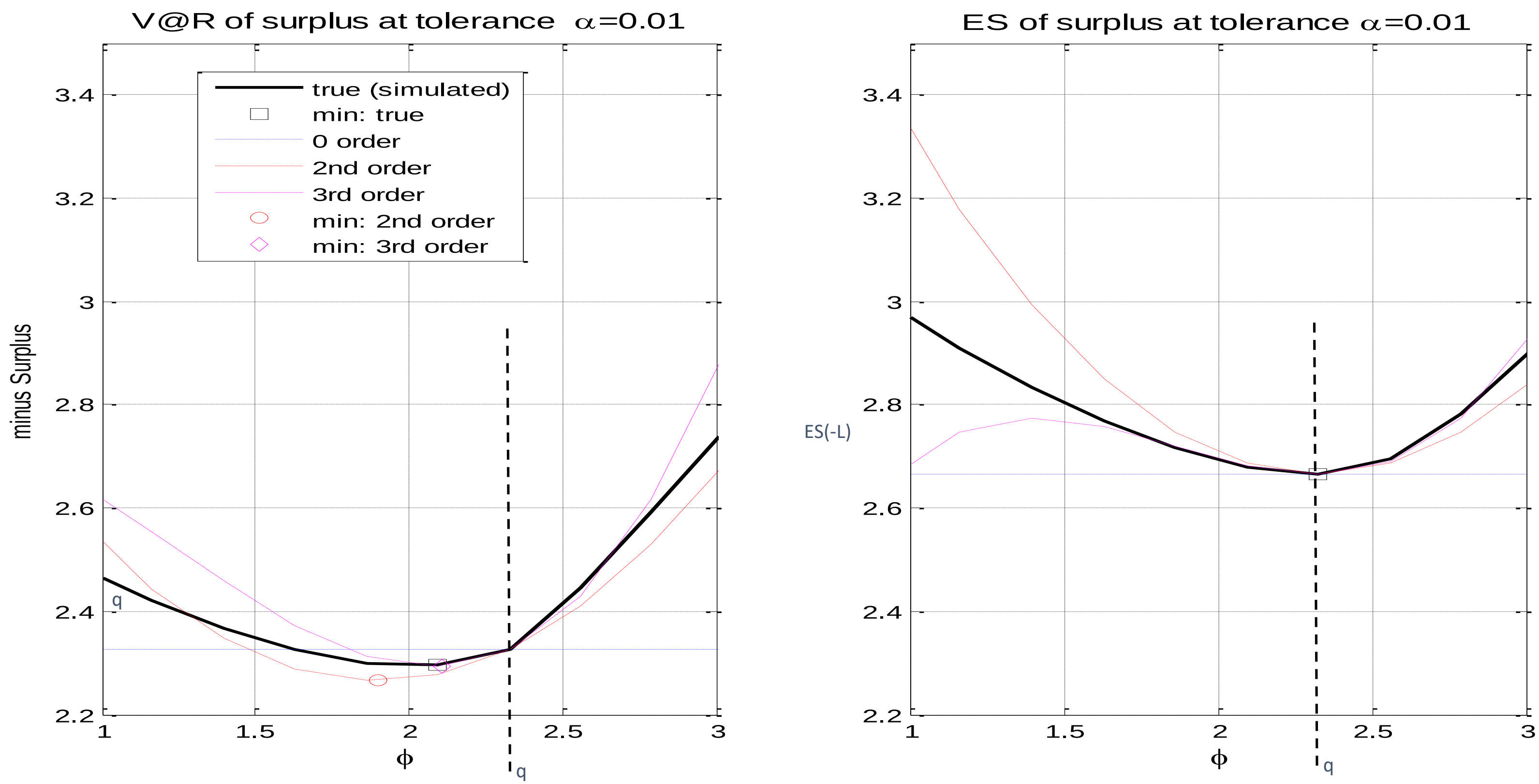


Figure 1: Value-at-risk $\mathrm{VaR}_\alpha[\mathrm{S}]$ (left) and expected shortfall $\mathrm{ES}_\alpha[\mathrm{S}]$ (right) as a function of the units ϕ of the financial asset X. The risk tolerance is set to $1-\alpha = 99.5\%$, the non-hedgeable component L is normally distributed with $\sigma_L = 0.388$ such that $q = \mathrm{VaR}_\alpha(\text{-L}) = 1$, and $\log(X)$ is log-normally distributed such that X has log-normal volatility $\sigma = 0.2$ and log-normal skew $\mu_3 = -0.3$.

Expansion results up to 3rd order coincide in good approximation with numerical findings.

Numerical analysis vs. theoretical findings

An extreme asset volatility and skew case



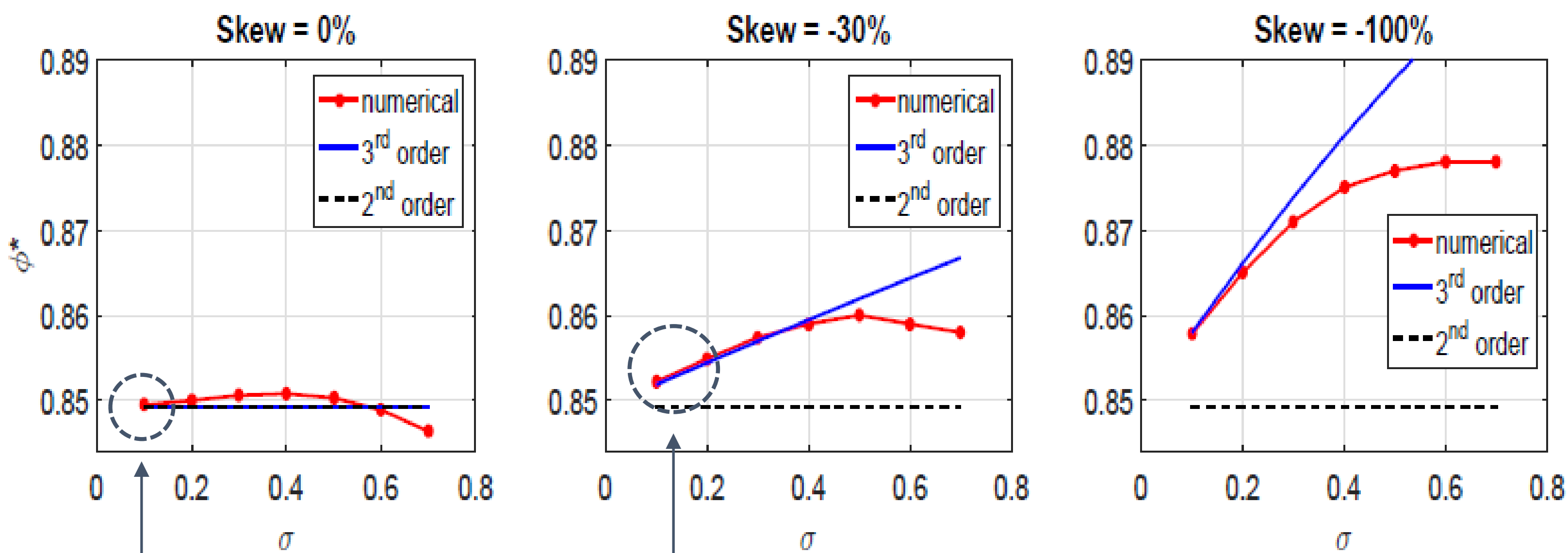
*) # simulations = 1e8, L ~N(0,1), X ~ Black Karasinski, i.e. ~ exp[- 20 * 0.05 * exp[N(-0.5^2/2,0.5)],
Standard deviation and skewness of log(X) amount to 0.53 and -1.76, respectively.

Even in extreme volatility and skew case expansion results up to 3rd order are pretty accurate around the optimum

Numerical analysis vs. theoretical findings

Location of the minimum

Risk-minimal investment amount ϕ^* for VaR99.5% as function of the log-normal volatility of X



Typical regime for FX risk (X ~ log-normal)

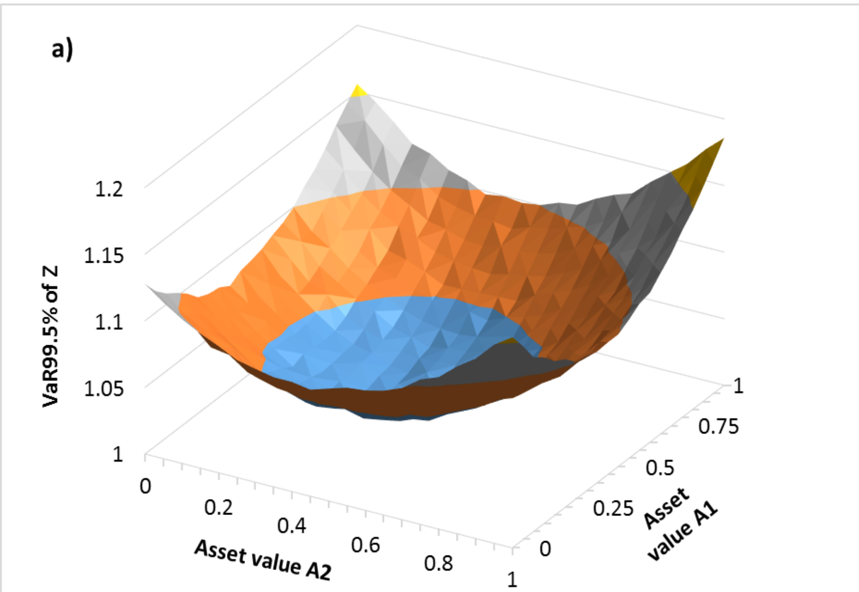
Typical regime for interest rate risk (X ~ log-log-normal)

Expansion results predict the features of the optimum very well for realistic parameter settings of FX and interest rate risk in a typical insurance portfolio.

Comparison with numerical results

Two normally distributed uncorrelated claim sizes*

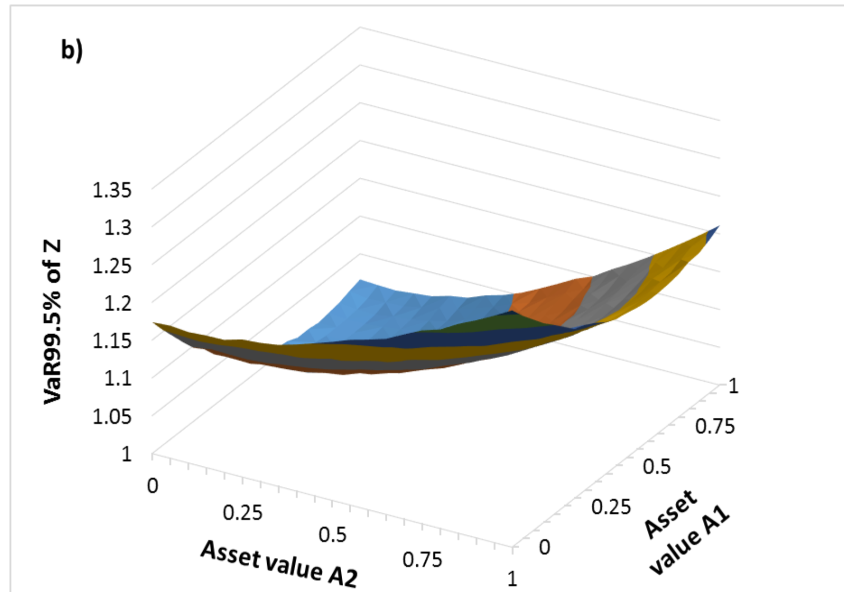
Symmetric case: $\sigma_1^L = \sigma_2^L = 0.275$



- **Optimum from theory:**

$\phi_1^* = \phi_2^* = 0.425$

Asymmetric case: $\sigma_1^L = 0.375,\ \sigma_2^L = 0.1$



- **Optimum from theory:**

$\phi_1^* = 0.79 \qquad \phi_2^* = 0.06$

Numerical results agree with the theory also for high market risk volatility

* $L_i \sim \mathcal{N}(0, \sigma_i^L)$, $X \sim \mathcal{LN}(\mu, \sigma_x)$ with $\sigma_x = 0.3$ and $\mu = -\frac{\sigma_x^2}{2}$,

Recipe for Construction of the ENP

For Value-at-Risk and Expected Shortfall based regimes

Expected-Shortfall based (SST)

- **Market value of liability**: replicate financial characteristics X_i (duration, currency, …) of *best-estimate liabilities* [+ *risk margin*]
- **Surplus structure**:
  a) Calculate $VaR_{\alpha=1\%}[Total\ Insurance\ Risk]$, i.e. all market factors fixed,
  b) allocate this risk figures to different financial factors X_i (using your favorite allocation method) and replicate these amounts accordingly
- **Free surplus**: Allocate remaining capital to risk-free investment (SFR cash)
- **Market risk component**: $ES_{\alpha=1\%}[Actuall\ Assets\ vs.ENP]$

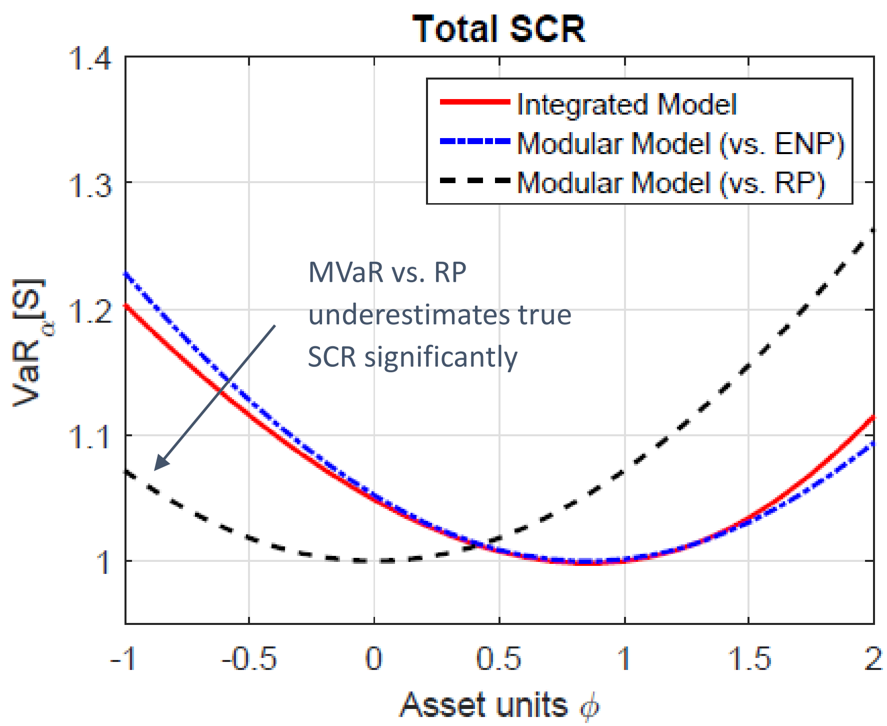
Value-at-risk based (Solvency II)

- **Market value of liability**: same as SST
- **Surplus structure**:
  a) Calculate $VaR_{1-\alpha=99.5\%}[Total\ Insur\ Risk]$
  b) Apply reduction factor ϕ_0^*/q (equals 85% if Insurance Risk normally distributed)
  c) [Increase this factor if assets exhibit significant negative skew]
  d) allocate adjusted total insurance risk to different financial factors X_i (using your favorite allocation method) and replicate these amounts accordingly
- **Free surplus**: Allocate remaining capital to risk-free investment (EUR cash)
- **Market risk component**: $VaR_{99.5\%}[Actuall\ Assets\ vs.ENP]$

Comparison of joint model with modular approach

Simple case with one liability cash flow

Total SCR for modular and integrated risk model



Model Calibration

- Surplus: $S(\phi) = \phi \cdot (1 - \mathrm{X}) + \mathrm{L}\,\mathrm{X}$
- X and L are assumed to be independent and normally distributed with:
  - X: std = 15%, mean = 1
  - L: std = 39%, mean = 0 → $SCR_L = 1$
- Modular Model: Aggregation to Total SCR is performed by means of the square root formula*:

$$SCR_T = \sqrt{SCR_L^2 + SCR_M^2}$$

- Market SCR_M calculated on mismatch:

$$S(\phi) = (\phi - \phi^*) \cdot X - \phi$$

- ϕ^* = 0.85 for ENP and ϕ^* = 0 for RP

Market risk measurement vs. the ENP leads to a total SCR in the modular model, which matches the total SCR of the integrated model very well

*Aggregation based on the square root formula is not fully adequate, because the total P&L is not normally distributed

Summary

- If you use a modular approach for Required Capital measurement, choose carefully the Neutral Position, i.e. the zero-risk asset portfolio in the market risk module.
- The Neutral Position replicates not exclusively the best-estimate liabilities. It must coincide with the risk minimal asset allocation in the integrated approach that models jointly market and insurance risks
- Otherwise (-> SII Standard Formula), the modular capital model might misestimate market risk significantly and give wrong ALM incentives
- We demonstrated that the Economic Neutral Position (ENP) is fairly model independent and can be implemented easily
    - For Expected Shortfall based Required Capital measurement, the ENP is given by replicating the market value of liability plus the Value-at-Risk of the insurance risk component.
    - For Value-at-Risk based Required Capital measurement, we provide approximations of the ENP that fit extreme well for realistic asset parameters.