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Hierarchical Compartmental Reserving Models

Insurance Capital Markets Research

Markus Gesmann

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Innovate - Differentiate - Scale

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Motivation

READY! FIRE! AIM!

VS.

AIM! READY! FIRE!

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Agenda

- Introduction to compartmental reserving modelling framework
- Modelling the mean claims process
- Modelling reserve uncertainty
- Case study using R/Stan & ‘brms’
- Summary

The compartmental reserving modelling framework

- Key idea: Start by fitting model to data, not data to model
- At the centre of the framework is to think about the data generating process
  - Begin by simulating artificial data that shares the expected real data characteristics
- Use "compartments" to reflect different business processes
  - Exposure being underwritten
  - Claims being reported
  - Payments being made

The compartmental reserving modelling framework

- Expert knowledge required to model and parameterise
  - **A Framework *not* a Method!**
- Benefits:
  - Transparent model that can be criticised
  - Provides additional insight into business processes
  - Practitioner knowledge can be incorporated into model easily

Relation to other models / frameworks

Chain-ladder methods	Cumulative growth curves	Incremental growth curves	Hierarchical compartmental model
Non-parametric	Parametric	Parametric	Parametric
Multiplicative	Multiplicative	Additive	Additive
Cumulative triangles	Cumulative triangles	Incremental triangles	Incremental triangles
Paid or incurred	Paid or incurred	Paid or incurred	Paid + Outstanding simultaneously
Mean + StdError	Mean + StdError (Max. Likli.)		Mean + StdError (Max. Likli.)
	Full distribution (Bayes)		Full distribution (Bayes)
Extensions BF Methods Munich-chain-ladder Bayes chain-ladder	Tail extrapolation		
Fairly robust, but projections on paid and incurred data can differ			Consistent with paid and outstanding data

Compartmental reserving models in a nutshell

- Use differential equations to model the claims process through time
- Consider which data generating distribution gave rise to the mean process, e.g. Gaussian, Log-normal, Negative-binomial, Tweedie
  - Which variance metric can be considered constant across claims development periods, if any? E.g. coefficient of variation
- Use expert knowledge to set priors on parameters
- Generate data from model: do simulations capture expected features?
- Update model with actual observations to obtain posterior parameter estimates and predictive distributions

Modelling mean claims process

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Compartmental models

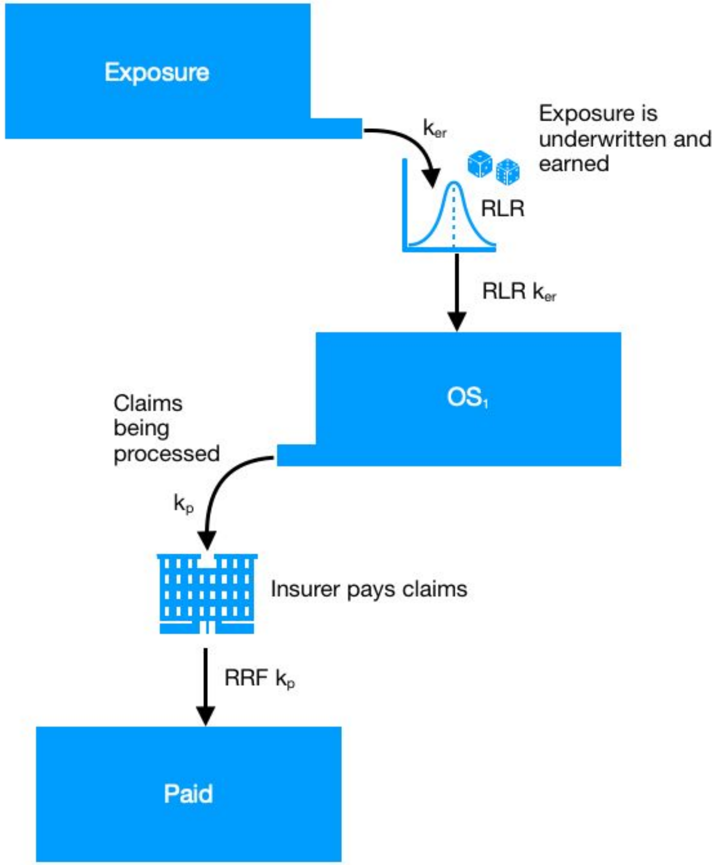
- Popular tool in multiple disciplines to describe the behaviour and dynamics of interacting processes using differential equations
- Each compartment relates to a different stage or population of the process, usually modelled with its own differential equation
- Examples are found in:
  - Pharma, to model how drugs interact with the body
  - Electric engineering, to describe the flow of electricity
  - Biophysics, to explain the interactions of neurons
  - Epidemiology, to understand the spread of diseases
  - Biology, to describe interaction of different populations

Simple Compartmental claims development model

$$dEX/dt = -k_{er} \cdot EX$$
$$dOS/dt = k_{er} \cdot RLR \cdot EX - k_p \cdot OS$$
$$dPD/dt = k_p \cdot RRF \cdot OS$$

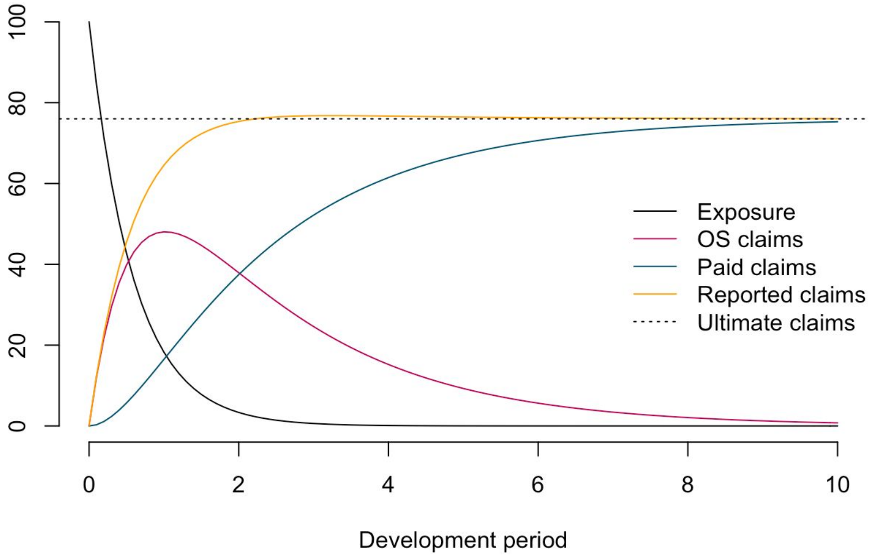
The parameters describe:

- k_{er} the rate at which claim events occur and are subsequently reported to the insurer
- RLR the reported loss ratio
- RRF the reserve robustness factor, the proportion of outstanding claims that eventually are paid
- k_p the rate of payment, i.e. the rate at which outstanding claims are paid



Analytical solutions can be derived by integration

Solutions Define Development Patterns



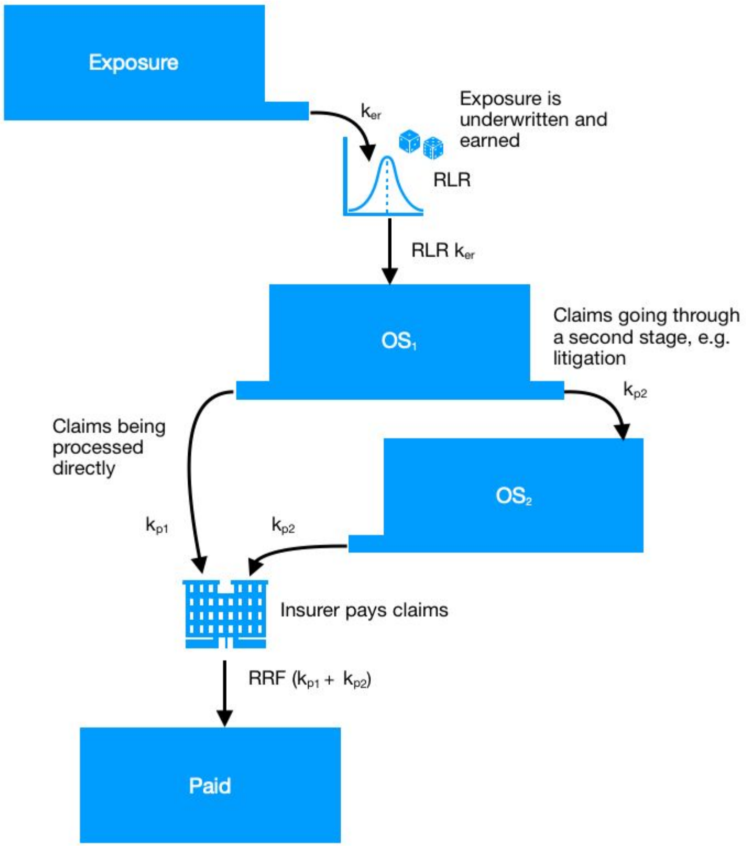
$$dEX/dt = -k_{er} \cdot EX$$
$$dOS/dt = k_{er} \cdot RLR \cdot EX - k_p \cdot OS$$
$$dPD/dt = k_p \cdot RRF \cdot OS$$

$$EX(t) = \Pi \cdot \exp(-k_{er} t)$$
$$OS(t) = \frac{\Pi \cdot RLR \cdot k_{er}}{k_{er} - k_p} \cdot (\exp(-k_p t) - \exp(-k_{er} t))$$
$$PD(t) = \frac{\Pi \cdot RLR \cdot RRF}{k_{er} - k_p}(k_{er} \cdot (1 - \exp(-k_p t) - k_p \cdot (1 - \exp(-k_{er} t))$$

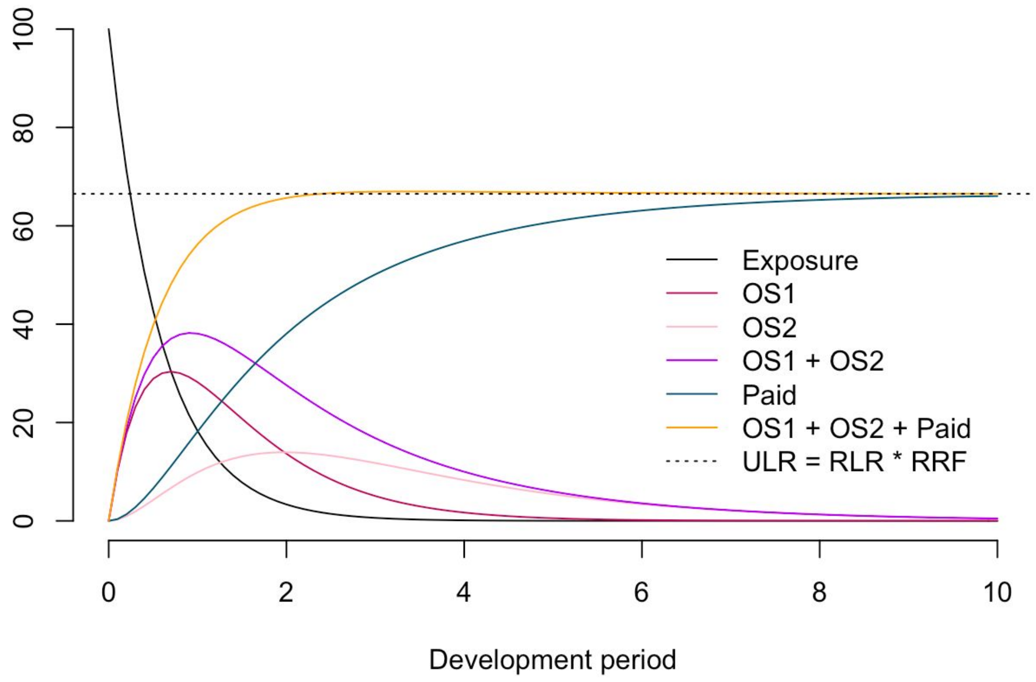
Compartmental model with two claims settlement processes

Single rate of settlement can be too simplistic to capture heterogeneous claims characteristics and hence settlement processes at an aggregated level

$$dEX/dt = -k_{er} \cdot EX$$
$$dOS_1/dt = k_{er} \cdot RLR \cdot EX - (k_{p_1} + k_{p_2}) \cdot OS_1$$
$$dOS_2/dt = k_{p_2} \cdot (OS_1 - OS_2)$$
$$dPD/dt = RRF \cdot (k_{p_1} \cdot OS_1 + k_{p_2} \cdot OS_2)$$



Analytical solutions illustrate different processes



$$
\begin{aligned}
EX(t) &= \Pi \, \exp\left(-k_{er} t\right) \\
OS_1(t) &= \frac{\Pi \, RLR \, k_{er}}{k_{er} - k_{p_1} - k_{p_2}} \left[\exp\left(-(k_{p_1} + k_{p_2})t\right) - \exp\left(-k_{er} t\right)\right] \\
OS_2(t) &= \frac{\Pi \, RLR \, k_{er} \, k_{p_2}}{k_{p_1}(k_{p_2} - k_{er})(k_{er} - k_{p_1} - k_{p_2})} \Big[\\
&\quad \exp\left(-(k_{p_1} + k_{p_2})t\right)(k_{er} - k_{p_2}) - \\
&\quad \exp\left(-k_{p_2} t\right)(k_{er} - k_{p_1} - k_{p_2}) - \\
&\quad \exp\left(-k_{er} t\right) k_{p_1} \Big] \\
PD(t) &= \frac{\Pi \, RLR \, RRF}{k_{p_1}(k_{p_2} - k_{er})(k_{er} - k_{p_1} - k_{p_2})} \Big[\\
&\quad \left(k_{p_1}\left(k_{er}\left(k_{p_1} - k_{er}\right) - k_{p_2}\left(k_{p_1} + k_{p_2}\right)\right) + 2 k_{er} k_{p_2}\right) + \\
&\quad \exp\left(-(k_{p_1} + k_{p_2})t\right)\left(k_{er}\left(k_{er} k_{p_1} - k_{er} k_{p_2} + k_{p_2}^2 - k_{p_1} k_{p_2}\right)\right) + \\
&\quad \exp\left(-k_{p_2} t\right)\left(k_{er} k_{p_2}\left(k_{er} - k_{p_1} - k_{p_2}\right)\right) + \\
&\quad \exp\left(-k_{er} t\right)\left(k_{p_1}\left(k_{p_1} k_{p_2} + k_{p_2}^2 - k_{er} k_{p_1}\right)\right) \Big]
\end{aligned}
$$

Compartment models can be extended easily …

- To incorporate different claims processes, e.g. a faster and slower settlement process
- Separate earning and reporting processes
- Time dependent parameters
- Calendar year effects

- Analytical solutions may become complex, but can opt for ODE solvers
- Note: <u>Paid claims are scaled integration of outstanding claims</u>

Modelling uncertainty

Be careful with your parameter bookkeeping

- In a Bayesian framework we distinguish between:
  - **Priors**, before we have actual data:
    - Prior parameter distribution, e.g. Planning Loss Ratio (PLR)
    - Prior predictive distribution, e.g. Capital Model Loss Ratio (CLR)
  - **Posteriors**, priors updated with actual data:
    - Posterior parameter distribution, e.g. Expected Loss Ratio (ELR)
    - Posterior predictive distribution, e.g. Ultimate Loss Ratio (ULR)

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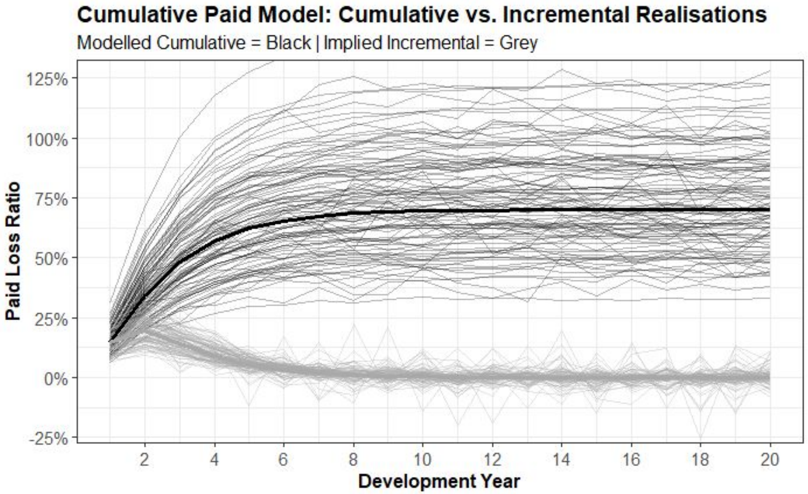
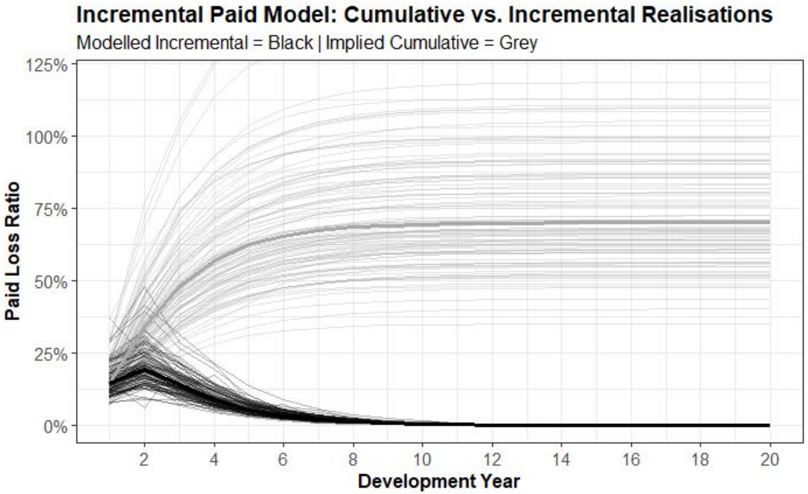
Which process variance metric can be kept constant?

Simulated Behaviour: Cumulative vs. Incremental Model





- Modelling cumulative paid data directly is problematic
- Modelling incremental paid with constant CoV more realistic

Parameter Uncertainty + Data Generating Process

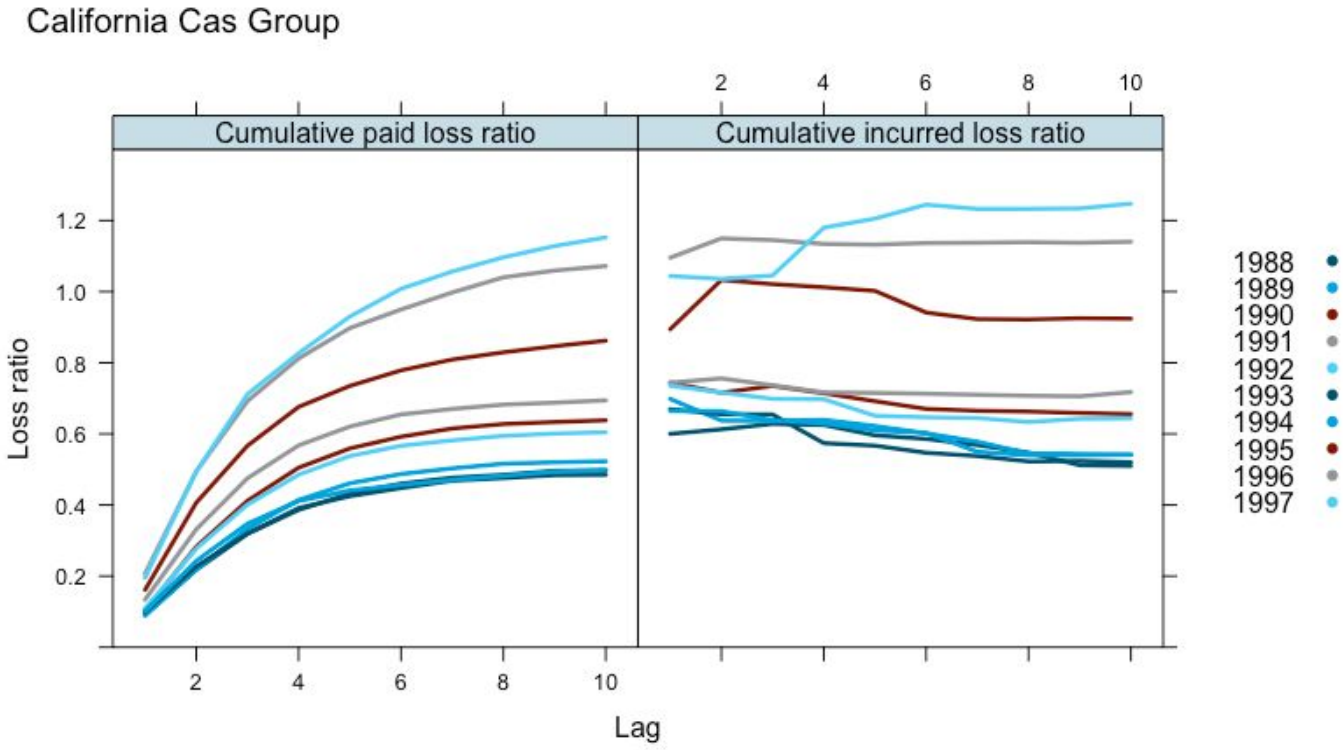
- "Which combinations are consistent with our data and model?"
  - Start with prior assumptions, e.g. ULR ~ logN(μ,σ), …
  - Update prior assumptions via the likelihood, L(y; ULR, ….)
  - Obtain 'posterior' parameter distributions, p(ULR, …|y)
- From posterior ELR to posterior ULR:
  1. Simulate realisations from posterior parameter distributions
  2. Simulate realisations from assumed data generation distribution
  3. Sum future paid increment posterior predictive paths

'Borrow Strength' with Hierarchies

- Which parameters vary across different cohorts, e.g. accident years and which are more likely to be fixed?
  - Chain-ladder assumption: shape of curves considered fixed across accident years
  - Ultimate loss ratios vary by accident years
- Hierarchical models allow all parameters to vary across cohort
  - A parameter has greater potential to deviate from the 'cohort average' parameter value where data are rich (credibility weighting / shrinkage)
  - Hierarchical priors are used to prevent overfitting (regularization)

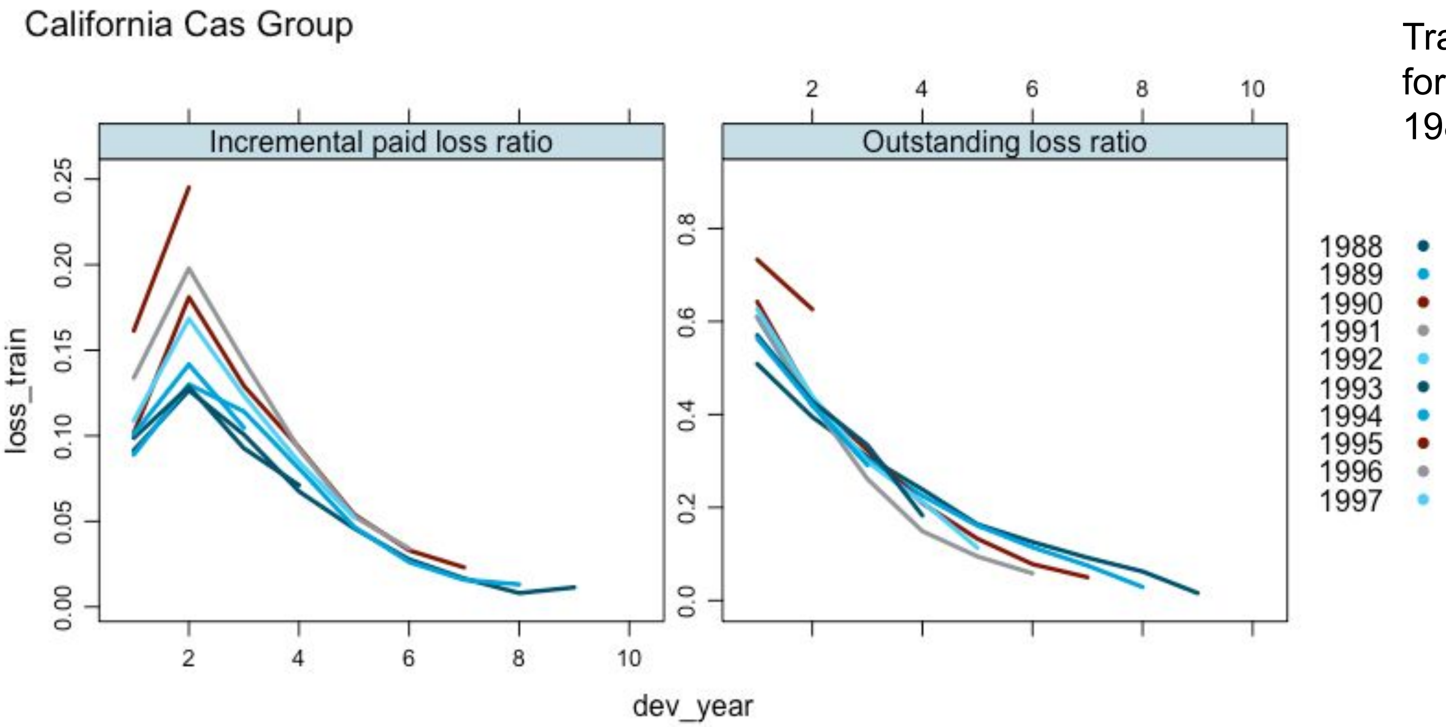
Case Study

Example data set: Cumulative paid and incurred



Full 10 year history for accident years 1988 - 1997

Example data set: Incremental paid and outstanding



Training triangles for accident years 1988 - 1997

Model process and location parameter

- Let *t* be the development period
- *y(t, δ)* describing paid (δ = 1) and outstanding claims (δ = 0)
- Assume process follows a log-normal distribution, with constant CoV$_\delta$
- We model the median of the claims process as:

$$
\begin{aligned}
y(t_j) &\sim \log \mathcal{N}(\mu(t_j), \sigma_\delta^2) \\
\mu(t_j) &= \log f(t_j; \Theta, \delta) \\
f(t_j; \Theta, \delta) &= (1 - \delta) OS(t_j; \Theta) + \delta\, (PD(t_j; \Theta) - PD(t_{j-1}; \Theta)) \\
\delta &= \begin{cases} 0 \text{ if } y \text{ is outstanding claim} \\ 1 \text{ if } y \text{ is paid claim} \end{cases}
\end{aligned}
$$

Setup analytical solution in Stan/C

```
myFuns <- "

real paid(real t, real ker, real kp, real RLR, real RRF){

 return(RLR*RRF/(ker - kp) * (ker *(1 - exp(-kp*t)) - kp*(1 -
exp(-ker*t))));

}


real os(real t, real ker, real kp, real RLR){

 return((RLR*ker/(ker - kp) * (exp(-kp*t) - exp(-ker*t))));

}
```

Setup analytical solution in Stan/C cont'd

```
real claimsprocess(real t, real devfreq, real oker, real okp,
                   real oRLR, real oRRF, real delta){
    real out;
    real ker = 1 + exp(oker); real kp = 1 * exp(okp * 0.5);
    real RLR = 0.7 * exp(oRLR * 0.1); real RRF = exp(oRLR * 0.1);

    out = os(t, ker, kp, RLR) * (1 - delta) + paid(t, ker, kp, RLR, RRF) * delta;
    if( (delta > 0) && (t > devfreq) ){ // paid greater dev period 1
     out = out - paid(t - devfreq, ker, kp, RLR, RRF)*delta;
    }
    return(out);
}"
```

Parameter structures

- Parameters assumed ‘fixed’ across accident years
  - $k_{er}\,,\; k_p\,,\; \sigma_{pd}\,,\; \sigma_{os}$
- Parameters assumed to vary ‘randomly’ by accident year
  - $RLR_{[i]}\,,\; RRF_{[i]}$ allowing for correlation:

$$\begin{pmatrix} RLR_{[i]} \\ RRF_{[i]} \end{pmatrix} \sim \begin{pmatrix} RLR_0 \\ RRF_0 \end{pmatrix} + \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right)$$

with RLR_0, RRF_0 following a log-normal distribution

Create non-linear model formula in R

```
frml <- bf(loss_train ~ log(claimsprocess(dev_year, 1.0, oker,
                                okp,oRLR, oRRF, delta)),
     oRLR ~ 1 + (1 |ID| origin_year),
     oRRF ~ 1 + (1 |ID| origin_year),
     oker ~ 1, okp ~ 1,
     sigma ~ 0 + deltaf,
     nl = TRUE)
```

Set prior parameter distributions

- Setting sensible priors is crucial for MCMC simulations
- Using standard Gaussians seems to be advisable
  - Standard Gaussian can be transformed to appropriate value ranges
- Example:
  - original oRLR ~ N(0, 1)
  - Transformed RLR = 0.7 * exp(oRLR * 0.1), i.e. log-normal distribution

Set prior distribution over parameters

```
mypriors <- c(prior(normal(0, 1), nlpar = "oRLR"),
              prior(normal(0, 1), nlpar = "oRRF"),
              prior(normal(0, 1), nlpar = "oker"),
              prior(normal(0, 1), nlpar = "okp"),
              prior(normal(-3, 0.2), class = "b",
                    coef="deltafpaid", dpar= "sigma"),
              prior(normal(-3, 0.2), class = "b",
                    coef="deltafos", dpar= "sigma"),
              prior(student_t(10, 0, 0.1), class = "sd", nlpar = "oRLR"),
              prior(student_t(10, 0, 0.05), class = "sd", nlpar = "oRRF"))
```

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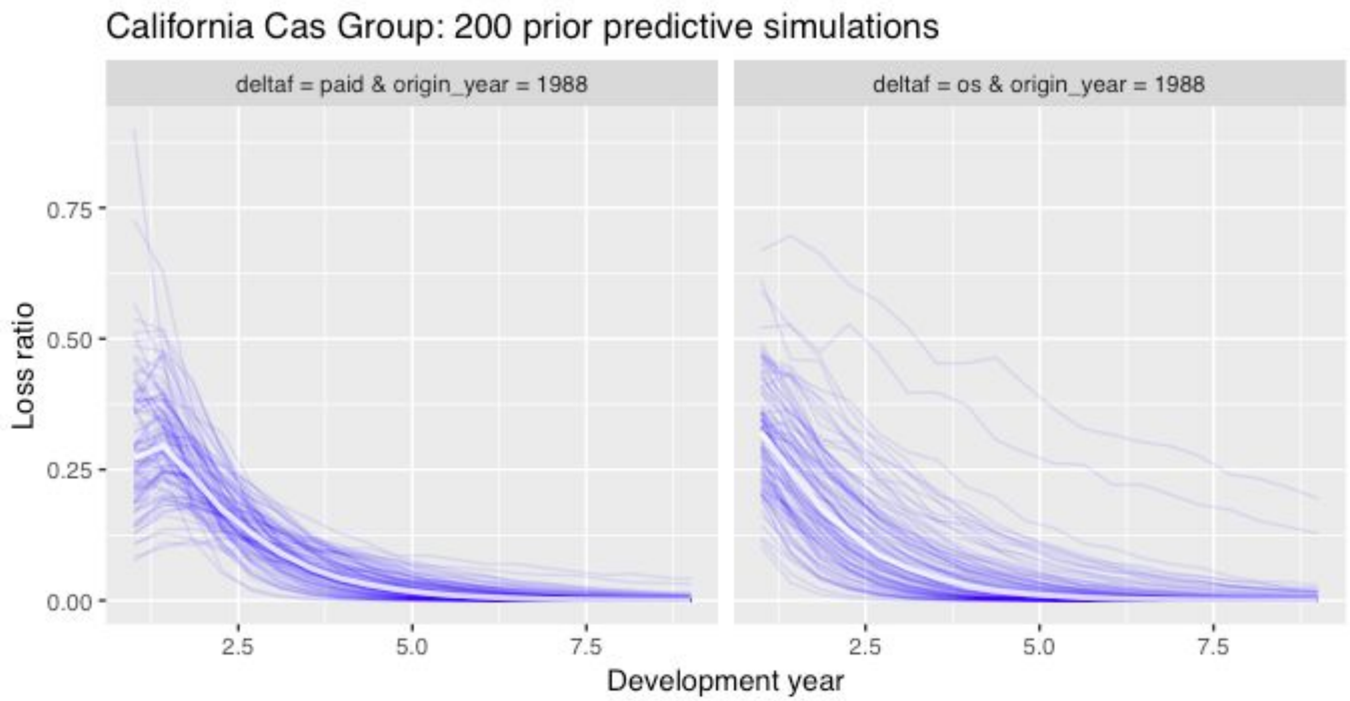
Run prior predictive model with ‘brms’ in R/Stan

```
b1a <- brm(frml, data = myDat,
  family = brmsfamily("lognormal", link_sigma = "log"),
  prior = mypriors,
  control = list(adapt_delta = 0.9, max_treedepth=15),
  file="models/CaliforniaGasLogNormalIncrPriorCGCana",
  stanvars = stanvar(scode = myFuns, block = "functions"),
  sample_prior = "only", seed = 123, iter = 200, chains = 2)
```

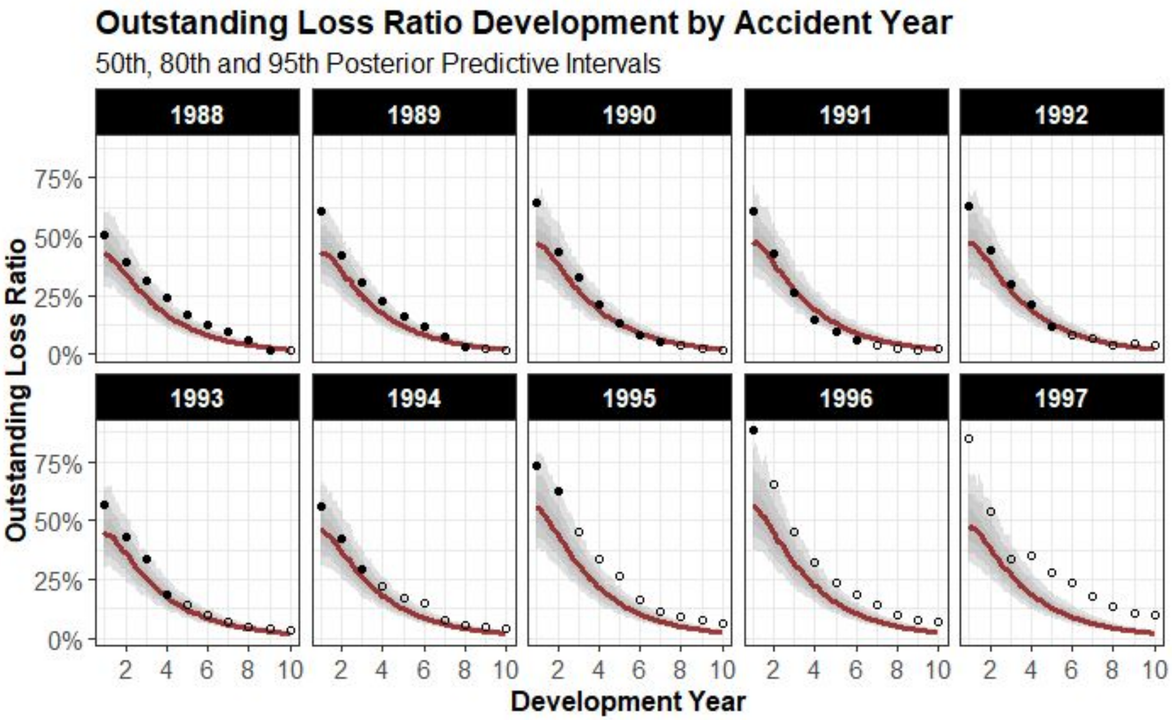
Review prior predictive output



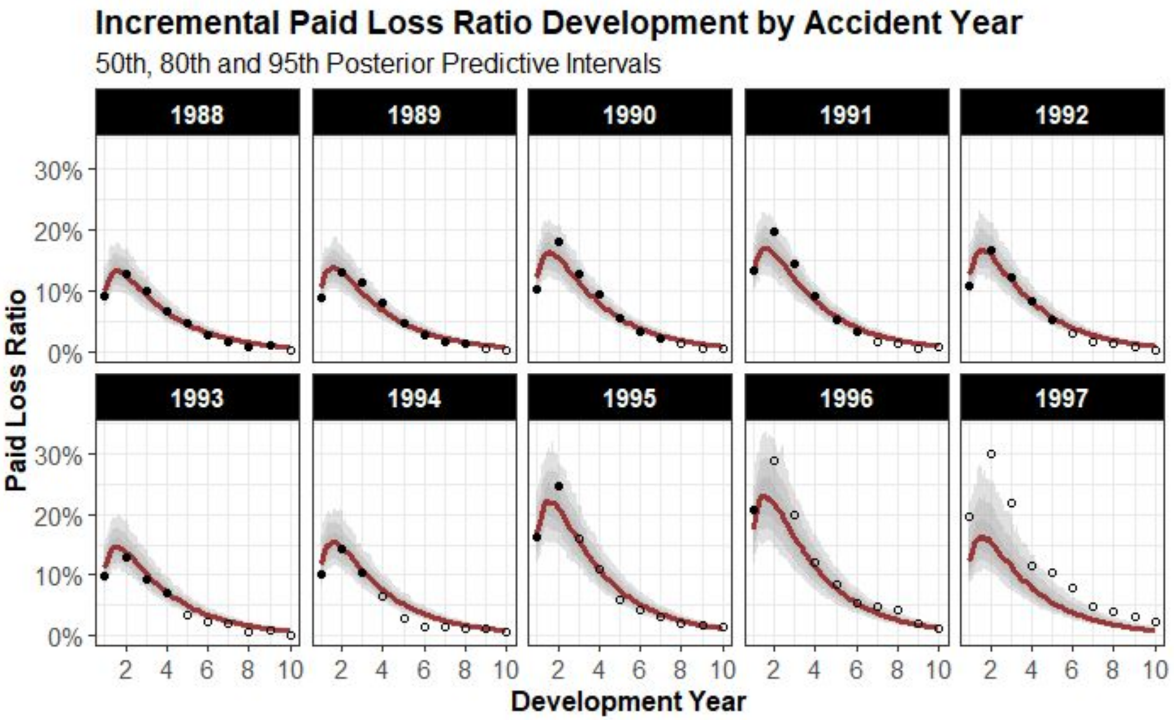
Run model with actual data

```
b1afit <- update(b1a, newdata=modDT_b[!is.na(loss_train)],
                 file="models/CaliforniaGasLogNormalIncrPosterior1CGCana",
                 sample_prior="no", seed=123, iter=500)
```

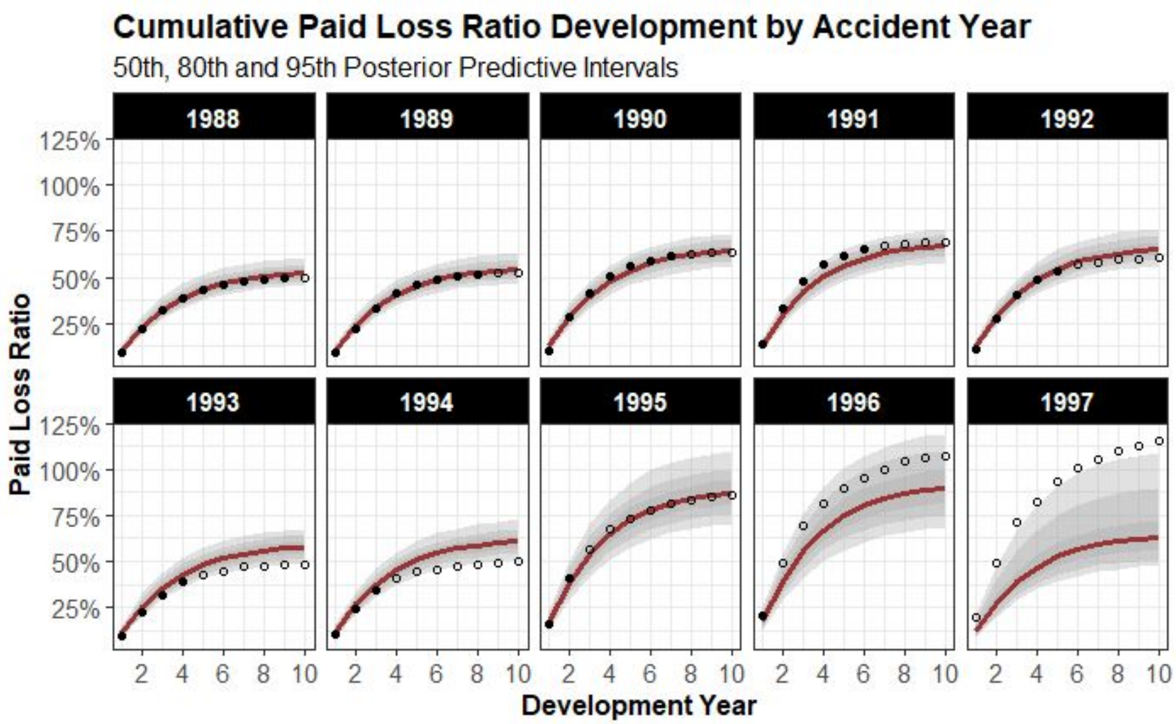
Outstanding data with holdouts



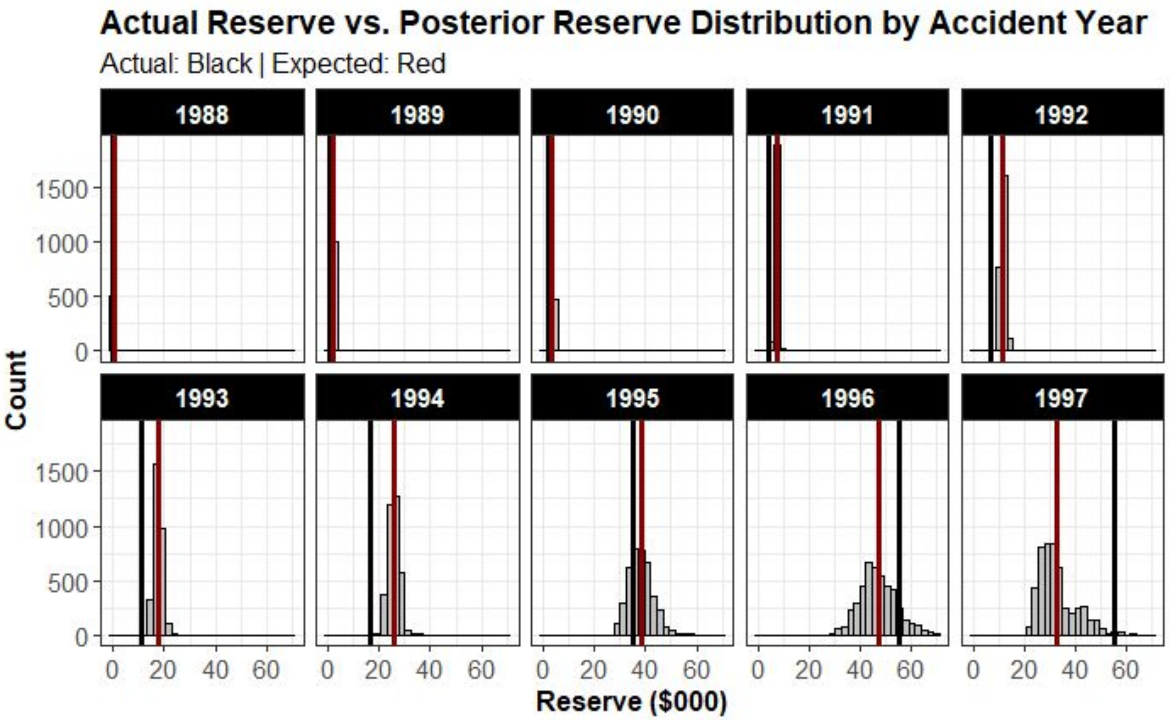
Paid data with holdouts



Cumulative Paid data with holdouts



Distribution of future payments



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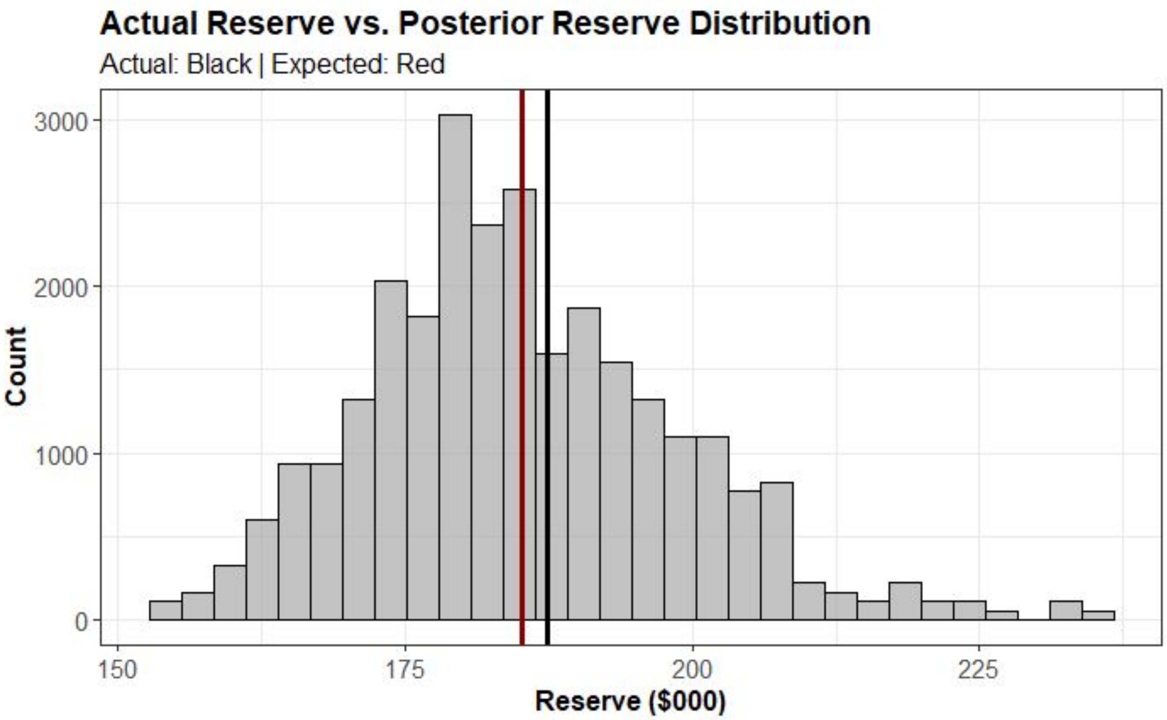
Distribution of future payments



From 500 samples:

Min.	151,776
1st Qu.	174,634
Median	183,273
Mean	184,184
3rd Qu.	191,748
Max.	236,694

Summary

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Summary

- Compartmental reserving models offer:
  - A flexible and transparent framework to develop parametric non-linear curves to describe development of outstanding and paid claims, simultaneously
  - Insight for a variety of small data sizes, as industry data and expert judgement can naturally be incorporated
  - Intuitive and transferable claims process-linked outputs
- Bayesian modelling framework offers flexible approach
  - Expert judgement required to set prior assumptions and review model output
- Paid development should be modelled on an incremental basis

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- See also: https://magesblog.com/categories/reserving/

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Insurance Capital Markets Research (ICMR) is a quantitative research consultancy that provides independent insights on the global specialty (re)insurance industry.

Our team of industry experts has a proven track record of delivering valuable research that helps clients make informed decisions.

We use our deep knowledge of the market to assess and model the performance and return profiles of (re)insurance entities and portfolios, both within Lloyd's and globally.

ICMR was founded by Lloyd's former heads of analysis and research who also worked together in the capital markets and insurance linked securities. ICMR was established in early 2020 and launched the RISX Index with Morningstar in 2021.

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The Founders

Markus Gesmann

Markus has spent 20 years in both insurance and capital markets. He is the former head of analysis at Lloyd's, where he set up a market wide analytical performance and price monitoring framework. Markus was head of pricing at an ILS joint venture with Lehman Brothers and Vario Partners, structuring innovative risk transfer solutions into capital markets.

Markus is an expert in modelling non-life insurance portfolios and probabilistic programming, and an Honorary Visiting Fellow at Bayes Business School, City, University of London.

E: markus@insurancecapitalmarkets.com

Quentin Moore

Quentin has over 30 years Lloyd's and capital markets experience, including directorships of managing agencies and head of research at Lloyd's where he co-authored Lloyd's Performance Management template in the aftermath of Lloyd's WTC losses of 2001, helping implement Lloyd's capital modelling and risk management.

Quentin co-founded an ILS joint venture with Lehman Brothers as well as co-founding Bermuda-based ILS firm, Vario Partners.
He has worked in insurance private equity, in investment banking and in actuarial consulting.

E: quentin@insurancecapitalmarkets.com

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